Number System 1

Key Concepts

1. Numbers 1, 2, 3.......∞, which are used for counting are called **natural numbers.** The collection of natural numbers is denoted by **N.** Therefore, N = {1, 2, 3, 4, 5, ...}.
2. When 0 is included with the natural numbers, then the new collection of numbers called is called **whole number**. The collection of whole numbers is denoted by **W**. Therefore, W = {0, 1, 2, 3, 4, 5, ...}.
3. The negative of natural numbers, 0 and the natural number together constitutes **integers**. The collection of integers is denoted by I. Therefore, I = {..., -3, -2, -1, 0, 1, 2, 3, ...}.
4. The numbers which can be represented in the form of p/q, where q ≠ 0 and p and q are integers are called **rational numbers**. Rational numbers are denoted by **Q**. If p and q are co-prime, then the rational number is in its simplest form.
5. All-natural numbers, whole numbers and integer are rational number.
6. **Equivalent rational numbers** (or fractions) have same (equal) values when written in the simplest form.
7. Rational number between two numbers x and y = $\frac{x+y}{2}$.
8. There are infinitely many rational numbers between any two given rational numbers.
9. The numbers which are not of the form of p/ q, where q ≠ 0 and p and q are integers are called irrational numbers. For example: $\sqrt{2}, \sqrt{7}$, π, etc.
10. Rational and irrational numbers together constitute are called **real numbers.** The collection of real numbers is denoted by **R.**
11. Irrational number between two numbers x and y

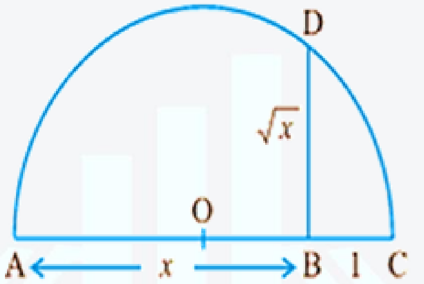
$$= \begin{cases} \sqrt{xy}, \text{if x and y both are irrational numbers} \\ \sqrt{xy}, \text{if x is rational number and y is irrational number} \\ \sqrt{xy}, \text{if } x \times y \text{ is not a perfect square and x, y both are rational numbers} \end{cases}$$

12. **Terminating fractions** are the fractions which leaves remainder 0 on division.
13. **Recurring fractions** are the fractions which never leave a remainder 0 on division.
14. The decimal expansion of **rational** number is **either terminating or non-terminating recurring**. Also, a number whose decimal expansion is terminating or non-terminating recurring is rational.
15. The decimal expansion of an **irrational** number is **non-terminating non-recurring.** Also, a number whose decimal expansion is non-terminating non-recurring is irrational.
16. Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.
17. The process of visualization of numbers on the number line through a magnifying glass is known as the process of **successive magnification.** This technique is used to represent a real number with non-terminating recurring decimal expansion.
18. Irrational numbers like $\sqrt{2}, \sqrt{3}, \sqrt{5} ... \sqrt{n}$, for any positive integer n can be represented on number line by using Pythagoras theorem.

19. If $a > 0$ is a real number, then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$.
20. For any positive real number x, we have:

$$x=\left(\frac{x+1}{2}\right)^2-\left(\frac{x+1}{2}\right)^2$$

21. For every positive real number x, $\sqrt{x}$ can be represented by a point on the number line using the following steps:
    i. Obtain the positive real number, say x.
    ii. Draw a line and mark a point A on it.
    iii. Mark a point B on the line such that AB = x units.
    iv. From B, mark a distance of 1 unit on extended AB and name the new point as C.
    v. Find the mid-point of AC and name that point as O.
    vi. Draw a semi-circle with centre O and radius OC.
    vii. Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D.
    viii. Length BD is equal to $\sqrt{x}$.



22. Properties of irrational numbers:
    i. The sum, difference, product and quotient of two irrational numbers need not always be an irrational number.
    ii. Negative of an irrational number is an irrational number.
    iii. Sum of a rational and an irrational number is irrational.
    iv. Product and quotient of a non-zero rational and irrational number is always irrational.
23. Let $a > 0$ be a real number and n be a positive integer. Then $\sqrt[n]{a} = b$, if $b^n = a$ and $b > 0$.

    The symbol '$\sqrt{\ }$' is called the **radical sign**.
24. For real numbers $a > 0$ and $b > 0$:
    i. $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$
    ii. $\sqrt{\frac{a}{b}}=\sqrt{\frac{b}{a}}$
    iii. $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b$
    iv. $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}+\sqrt{d}\right)=\sqrt{ac}+\sqrt{bc}+\sqrt{ad}-\sqrt{bd}$
    v. $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=a^2-b$
    vi. $\left(\sqrt{a}+\sqrt{b}\right)^2=a+b+2\sqrt{ab}$
25. The process of removing the radical sign from the denominator of an expression to convert it to an equivalent expression whose denominator is a rational number is called **rationalising the denominator**.
26. The multiplicating factor used for rationalising the denominator is called the **rationalising factor**.

27. If a and b are positive real numbers, then

    Rationalising factor of $\frac{1}{\sqrt{a}}$ is $\sqrt{a}$

    Rationalising factor of $\frac{1}{a \pm \sqrt{b}}$ is $a \neq \sqrt{b}$

    Rationalising factor of $\frac{1}{\sqrt{a} \pm \sqrt{b}}$ is $\sqrt{a} \pm \sqrt{b}$

28. The **exponent** is the number of times the base is multiplied by itself.
29. In the exponential representation a^m, a is called the **base** and m is called the **exponent or power**.
30. **Laws of exponents:** If a, b are positive real numbers and m, n are rational numbers, then
    i. $a^m \times a^n = a^{m-n}$
    ii. $a^m \div a^n = a^{m-n}$
    iii. $(a^m)^n = a^{mn}$
    iv. $a^{-n} = \frac{1}{a^n}$
    v. $(ab)^n = a^n b^n$
    vi. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
    vii. $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$ or $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Numbers

Number: Arithmetical value representing a particular quantity. The various types of numbers are Natural Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, Real Numbers etc.

Natural Numbers

Natural numbers(N) are positive numbers i.e. 1, 2, 3 ..and so on.

Whole Numbers

Whole numbers (W) are 0, 1, 2, .. and so on. Whole numbers are all Natural Numbers including '0'. Whole numbers do not include any fractions, negative numbers or decimals.

Integers

Integers are the numbers that includes whole numbers along with the negative numbers.

Rational Numbers

A number 'r' is called a rational number if it can be written in the form p/q, where p and q are integers and q ≠ 0.

Irrational Numbers

Any number that cannot be expressed in the form of p/q, where p and q are integers and q≠0, is an irrational number. Examples: $\sqrt{2}$, 1.010024563..., e, π

Real Numbers

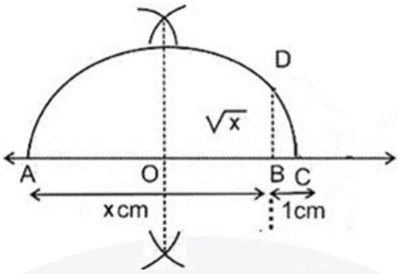
Any number which can be represented on the number line is a Real Number(R). It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

Irrational Numbers

Representation of Irrational numbers on the Number line

Let $\sqrt{x}$ be an irrational number. To represent it on the number line we will follow the following steps:

- Take any point A. Draw a line AB = x units.
- Extend AB to point C such that BC = 1 unit.
- Find out the mid-point of AC and name it 'O'. With 'O' as the centre draw a semi-circle with radius OC.
- Draw a straight line from B which is perpendicular to AC, such that it intersects the semi-circle at point D.
- Length of BD = $\sqrt{x}$.



Constructions to Find the root of x.

With BD as the radius and origin as the centre, cut the positive side of the number line to get $\sqrt{x}$.

Identities for Irrational Numbers

Arithmetic operations between:

- rational and irrational will give an irrational number.
- irrational and irrational will give a rational or irrational number.

Example: $2 \times \sqrt{3} = 2\sqrt{3}$ i.e. irrational. $\sqrt{3} \times \sqrt{3} = 3$ which is rational.

Identities for irrational numbers

Rationalisation

Rationalisation is converting an irrational number into a rational number. Suppose if we have to rationalise $1/\sqrt{a}$.

$1/\sqrt{a} \times 1/\sqrt{a} = 1/a$

Rationalisation of $1/\sqrt{a} + b$:

$(1/\sqrt{a} + b) \times (1/\sqrt{a} - b) = (1/a - b^2)$

Laws of Exponents for Real Numbers

If a, b, m and n are real numbers then:

$a^m \times a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$a^m/a^n = a^{m-n}$

$a^m b^m = (ab)^m$

Here, a and b are the bases and m and n are exponents.

Exponential representation of irrational numbers

If a > 0 and n is a positive integer, then: n $\sqrt{a}$ = a1n Let a > 0 be a real number and p and q be rational numbers, then:

$a^p \times a^q = a^{p+q}$

$(a^p)^q = a^{pq}$

$a^p / a^q = a^{p-q}$

$a^p b^p = (ab)^p$

Decimal Representation of Rational Numbers

Decimal expansion of Rational and Irrational Numbers

The decimal expansion of a rational number is either terminating or non- terminating and recurring.

Example: 1/2 = 0.5, 1/3 = 3.33.......

The decimal expansion of an irrational number is non terminating and non-recurring.

Examples: $\sqrt{2}$ = 1.41421356..

Expressing Decimals as rational numbers

Case 1 – Terminating Decimals

Example – 0.625

Let x = 0.625

If the number of digits after the decimal point is y, then multiply and divide the number by 10y.

So, x = 0.625 × 1000/1000 = 625/1000 Then, reduce the obtained fraction to its simplest form.

Hence, x = 5/8

Case 2: Recurring Decimals

If the number is non-terminating and recurring, then we will follow the following steps to convert it into a rational number:

Example = $1.0\overline{42}$

Step 1. Let x = $1.0\overline{42}$ ....(1)

Step 2. Multiply the first equation with 10y, where y is the number of digits that are recurring.

Thus, 100x = $104.\overline{242}$ ....(2)

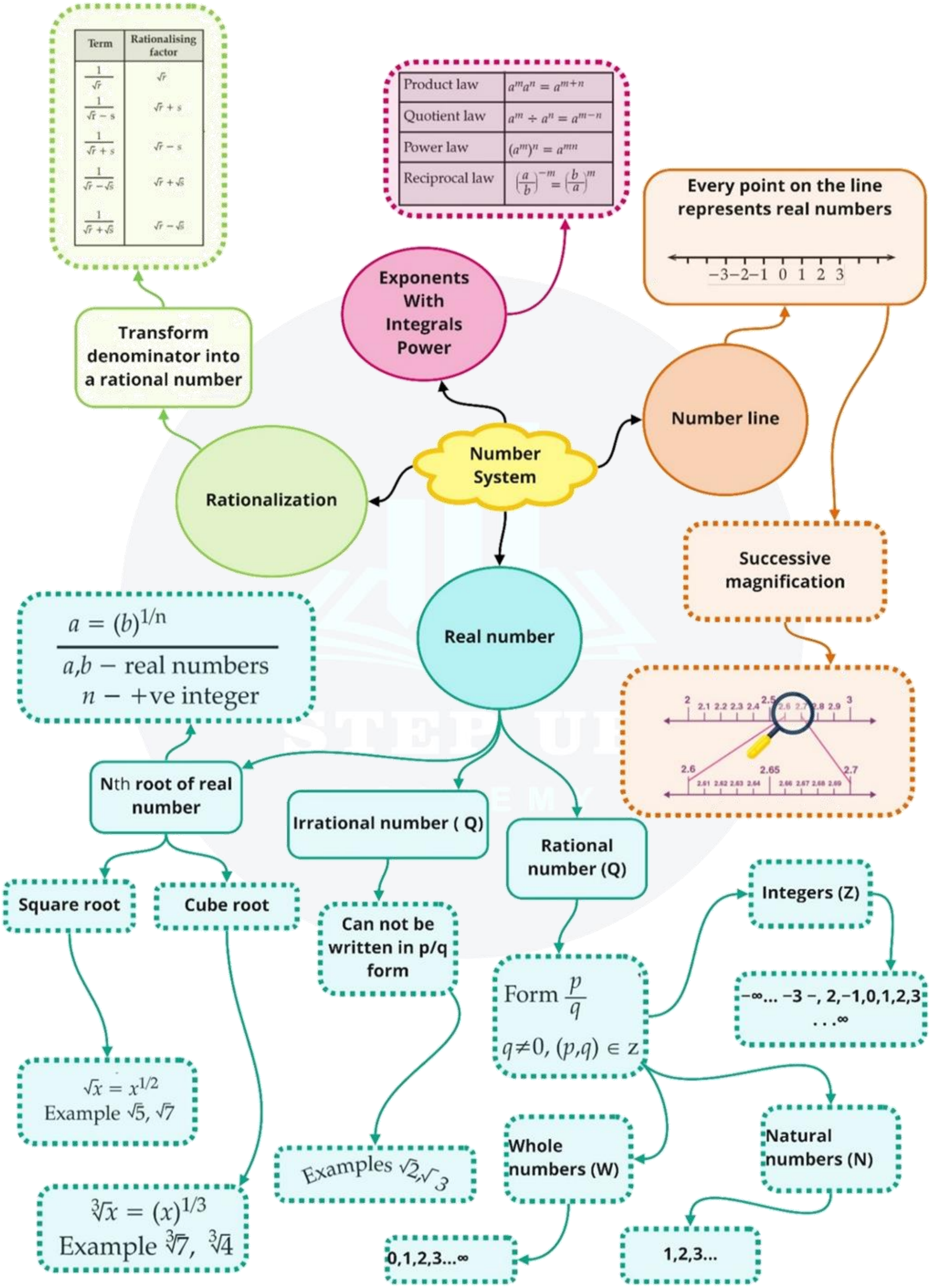
Steps 3. Subtract equation 1 from equation 2. On subtracting equation 1 from 2, we get

99x = 103.2x = 103.2/99 = 1032/990

Which is the required rational number.

Reduce the obtained rational number to its simplest form Thus, X = 172/165

Class : 9th mathematics
Chapter- 1: Number System



Important Questions

Multiple Choice Questions

1. Can we write 0 in the form of p/q?
   (a) Yes
   (b) No
   (c) Cannot be explained
   (d) None of the above
2. The three rational numbers between 3 and 4 are:
   (a) 5/2,6/2,7/2
   (b) 13/4,14/4,15/4
   (c) 12/7,13/7,14/7
   (d) 11/4,12/4,13/4
3. In between any two numbers there are:
   (a) Only one rational number
   (b) Many rational numbers
   (c) Infinite rational numbers
   (d) No rational number
4. Every rational number is:
   (a) Whole number
   (b) Natural number
   (c) Integer
   (d) Real number
5. $\sqrt{9}$ is a ________ number.
   (a) Rational
   (b) Irrational
   (c) Neither rational or irrational
   (d) None of the above
6. Which of the following is an irrational number?
   (a) $\sqrt{16}$
   (b) $\sqrt{(12/3)}$
   (c) $\sqrt{12}$
   (d) $\sqrt{100}$
7. $3\sqrt{6}+4\sqrt{6}$ is equal to:
   (a) $6\sqrt{6}$
   (b) $7\sqrt{6}$
   (c) $4\sqrt{12}$
   (d) $7\sqrt{12}$
8. $\sqrt{6}\times\sqrt{27}$ is equal to:
   (a) $9\sqrt{2}$
   (b) $3\sqrt{3}$
   (c) $2\sqrt{2}$
   (d) $9\sqrt{3}$
9. Which of the following is equal to x^3?
   (a) $x^6 - x^3$
   (b) $x^6 \cdot x^3$
   (c) x^6/x^3
   (d) $(x^6)^3$
10. Which of the following are irrational numbers?
   (a) $\sqrt{23}$
   (b) $\sqrt{225}$
   (c) 0.3796
   (d) 7.478478

Very Short Questions:

1. Simplify: $\left(\sqrt{5}+\sqrt{2}\right)^2$.
2. Find the value of $\sqrt{(3)^{-2}}$.
3. Identify a rational number among the following numbers:
4. Express 1.8181... in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0.
5. Simplify: $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$
6. Find the value of $\dfrac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}}$
7. Find the value of $\dfrac{4}{(216)^{\frac{-2}{3}}}-\dfrac{1}{(256)^{\frac{-3}{4}}}$

Short Questions:

1. Evaluate: $\left(\sqrt{5}+\sqrt{2}\right)^2+\left(\sqrt{8}-\sqrt{5}\right)^2$
2. Express 23.43 in $\frac{p}{q}$ Form, where p, q are integers and q ≠ 0.
3. Let 'a' be a non-zero rational number and 'b' be an irrational number. Is 'ab' necessarily an irrational? Justify your answer with example.

4. Let x and y be a rational and irrational numbers. Is x + y necessarily an irrational number? Give an example in support of your answer.
5. Represent $\sqrt{3}$ on the number line.
6. Represent $\sqrt{3.2}$ on the number line.
7. Express 1.32 + 0.35 as a fraction in the simplest form.

Long Questions:

1. If $x=\frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}$, then prove that $q^2 - 2px + 9 = 0$.
2. If $a=\frac{1}{3-\sqrt{11}}$ and $b=\frac{1}{a}$, then find $a^2 - b^2$.
3. Simplify $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\frac{2\sqrt{3}}{\sqrt{6}+2}$.
4. Prove that:
$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...+\frac{1}{\sqrt{8}+3}=2.$$
5. Find a and b, if
$$\frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{3}}+\frac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}}=a+\sqrt{15}b.$$

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion(A): 0.271 is a terminating decimal and we can express this number as 271/1000 which is of the form p/q, where p and q are integers and q ≠ 0.

   Reason (R): A terminating or non-terminating decimal expansion can be expressed as rational number.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion (A): Every integer is a rational number.

   Reason(R): Every integer 'm' can be expressed in the form m/1.

Answer Key

Multiple Choice Questions

1. (a) Yes
2. (b) 13/4,14/4,15/4
3. (c) Infinite rational numbers
4. (d) Real number
5. (a) Rational
6. (c) $\sqrt{12}$
7. (b) $7\sqrt{6}$
8. (a) $9\sqrt{2}$
9. (c) x^6/x^3
10. (a) $\sqrt{23}$

Very Short Answer:

1. Here, $\left(\sqrt{5}+\sqrt{22}\right)=\left(\sqrt{52}+2\sqrt{5}\sqrt{2}\right)+\left(\sqrt{2}\right)^2$

   = 5 + $2\sqrt{10}$ + 2 = 7 + $2\sqrt{10}$

2. $\sqrt{(3)^{-2}}=\left(3^{-2}\right)^{\frac{1}{2}}=3^{-2\times\frac{1}{2}}=3^{-1}=\frac{1}{3}.$

3. 0 is a rational number.

4. Let x =1.8181… …(i)

   100x = 181.8181……(ii) [multiplying eqn. (i) by 100]

   99x = 180 [subtracting (i) from (ii)]

   $x=\frac{180}{99}$

   Hence, 1.8181 … = $\frac{180}{99}=\frac{20}{11}$

5. $\sqrt{45}-3\sqrt{20}+4\sqrt{5}=3\sqrt{5}-6\sqrt{5}+4\sqrt{5}=\sqrt{5}.$

6. $\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}}=\frac{1-\frac{1}{0.1}}{\frac{8}{3}\times\frac{27}{8}+(-3)}$

   $=\frac{1-10}{9-3}=\frac{-9}{6}=-\frac{3}{2}$

7. $\frac{4}{(216)^{\frac{-2}{3}}}-\frac{1}{(256)^{\frac{-3}{4}}}=4\times(216)^{\frac{2}{3}}-(256)^{\frac{3}{4}}$

   $=4\times(6\times6\times6)^{\frac{2}{3}}-(4\times4\times4\times4)^{\frac{3}{4}}$

   $=4\times6^{3\times\frac{2}{3}}-4^{4\times\frac{3}{4}}=4\times6^2-4^3$

   = 4 × 36 – 64 = 144 – 64 = 80

Short Answer:

1. **Solution:**

   $\left(\sqrt{5}+\sqrt{2}\right)^2+\left(\sqrt{8}-\sqrt{5}\right)^2=5+2+2\sqrt{10}+8+5-2\sqrt{40}$

   $=20+2\sqrt{10}-4\sqrt{10}=20-2\sqrt{10}$

2. **Solution:**

   Let x = $23.\overline{43}$

   or x = 23.4343……(i)

   100x = 2343.4343……(ii) [Multiplying eqn. (i) by 100]

   99x = 2320 [Subtracting (i) from (ii)

   $\Rightarrow x=\frac{2320}{99}$

   Hence, $23.\overline{43}=\frac{2320}{99}$

3. **Solution:**

   Yes, 'ab' is necessarily an irrational.

   For example, let a = 2 (a rational number) and b = $\sqrt{2}$ (an irrational number)

   If possible let ab = 2 $\sqrt{2}$ is a rational number.

   Now $\frac{ab}{a}=\frac{2\sqrt{2}}{2}=\sqrt{2}$ is a rational number.

   [∵ The quotient of two non-zero rational number is a rational]

   But this contradicts the fact that $\sqrt{2}$ is an irrational number.

   Thus, our supposition is wrong.

   Hence, ab is an irrational number.

4. **Solution:**

   Yes, x + y is necessarily an irrational number.

   For example, let x = 3 (a rational number) and

   y = $\sqrt{5}$ (an irrational number)

   If possible, let x + y = 3 + $\sqrt{5}$ be a rational number.

   Consider $\frac{p}{q}=3+\sqrt{5}$, where p, q ∈ Z and q ≠ 0.

   Squaring both sides, we have

   $\frac{p^2}{q^2}=9+5+6\sqrt{5}\Rightarrow\frac{p^2}{q^2}=14+6\sqrt{5}$

   $\frac{p^2}{q^2}-14=6\sqrt{5}\Rightarrow\frac{p^2-14q^2}{6q^2}=\sqrt{5}$

$\because \frac{p}{q}$ is a rational $\Rightarrow \frac{p^2-14q^2}{6q^2}$ is a rational

$\because \frac{p}{q}$ is a rational

$\sqrt{5}$ is a rational

$\Rightarrow \sqrt{5}$ is a rational

But this contradicts the fact that $\sqrt{5}$ is an irrational number.

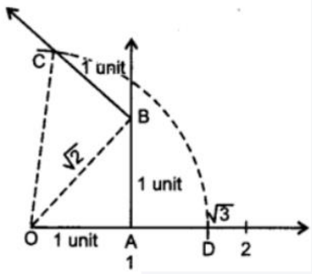
Thus, our supposition is wrong.

Hence, x + y is an irrational number.

5. **Solution:**

Here, $\sqrt{3}=\sqrt{1+2}=\sqrt{(1)^2+(\sqrt{2})^2}$

And, $\sqrt{2}=\sqrt{1+1}=\sqrt{(1)^2+(1)^2}$



On the number line, take OA = 1 unit.

Draw AB = 1 unit perpendicular to OA. Join OB.

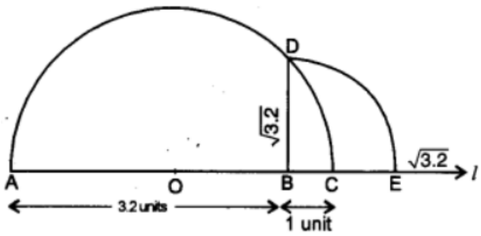
Again, on OB, draw BC = 1 unit perpendicular to OB. Join OC.

By Pythagoras Theorem, we obtain OC $=\sqrt{3}$. Using compasses, with Centre O and radius OC, draw an arc, which intersects the number line at point D. Thus, OD= $\sqrt{3}$ and D corresponds to $\sqrt{3}$.

6. **Solution:**

First of all draw a line of length 3.2 units such that AB = 3.2 units. Now, from point B, mark a distance of 1 unit. Let this point be 'C'. Let 'O' be the mid-point of the distance AC. Now, draw a semicircle with Centre 'O' and radius OC. Let us draw a line perpendicular to AC passing through the point 'B' and intersecting the semicircle at point 'D'.

∴ The distance BD = $\sqrt{3.2}$



Now, to represent $\sqrt{3.2}$ on the number line. Let us take the line BC as number line and point 'B' as zero, point 'C' as '1' and so on. Draw an arc with Centre B and radius BD, which intersects the number line at point 'E'.

Then, the point 'E' represents $\sqrt{3.2}$.

7. **Solution:**

Let. x = $1.3\overline{2}$ = 1.3222.... (i)

Multiplying eq. (i) by 10, we have

10x = 13.222.... (ii)

Again, multiplying eq. (i) by 100, we have

100x = 132.222... ...(iii)

Subtracting eq. (ii) from (iii), we have

100x – 10x = (132.222...) – (13.222...)

90x = 119

$\Rightarrow x=\frac{119}{90}$

Again, y = $0.\overline{35}$ = 0.353535......(iv)

Multiply (iv) by 100, we have ...(iv)

100y = 35.353535... (v)

Subtracting (iv) from (u), we have

100y – y = (35.353535...) – (0.353535...)

99y = 35

$\Rightarrow y=\frac{35}{99}$

Now, $1.3\overline{2}+0.\overline{35}=x+y=\frac{119}{90}+\frac{35}{99}$

$=\frac{1309+350}{90}=\frac{1659}{990}=\frac{553}{330}$

Long Answer:

1. **Solution:**

$x=\frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}$

$=\frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}\times\frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}}$

$=\frac{\left(\sqrt{p+q}+\sqrt{p-q}\right)^2}{\left(\sqrt{p+q}\right)^2-\left(\sqrt{p-q}\right)^2}$

$=\frac{p+q+p-q+2\times\sqrt{p+q}\times\sqrt{p-q}}{(p+q)-(p-q)}$

$=\frac{2p+2\sqrt{p^2-q^2}}{2q}=\frac{p+\sqrt{p^2-q^2}}{q}$

$\Rightarrow \quad qx = p + \sqrt{p^2 - q^2}$

$\Rightarrow qx - p = \sqrt{p^2 - q^2}$

Squaring both sides, we have

$\Rightarrow q^2x^2 + p^2 - 2pqx = p^2 - q^2$

$\Rightarrow q^2x^2 - 2pqx + q^2 = 0$

$\Rightarrow q(q^2 - 2px + q) = 0$

$\Rightarrow qx^2 - 2px + q = 0$ ($\because q \neq 0$)

2. Solution:

Here, $a = \dfrac{1}{3-\sqrt{11}} \times \dfrac{3+\sqrt{11}}{3+\sqrt{11}}$

$= \dfrac{3+\sqrt{11}}{9-11} = \dfrac{3+\sqrt{11}}{-2}$

$b = \dfrac{1}{a} = 3 - \sqrt{11}$

Now $a^2 - b^2 = (a + b)(a - b)$

$= \left(\dfrac{3+\sqrt{11}}{-2} + 3 - \sqrt{11}\right)\left(\dfrac{3+\sqrt{11}}{-2} - 3 + \sqrt{11}\right)$

$= \left(\dfrac{-3-\sqrt{11}+6-2\sqrt{11}}{2}\right)\left(\dfrac{-3-\sqrt{11}-6+2\sqrt{11}}{2}\right)$

$= \left(\dfrac{3-3\sqrt{11}}{2}\right)\left(\dfrac{-9+\sqrt{11}}{2}\right) = \dfrac{-27+3\sqrt{11}+27\sqrt{11}-33}{4}$

$\dfrac{-60+30\sqrt{11}}{4} = \dfrac{-30+15\sqrt{11}}{2} = \dfrac{1}{2}\left(15\sqrt{11}-30\right)$

3. Solution:

$\dfrac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \dfrac{2\sqrt{3}}{\sqrt{6}+2}$

$\dfrac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} \times \dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} - \dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} \times \dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$

$+ \dfrac{2\sqrt{3}}{\sqrt{6}+2} \times \dfrac{\sqrt{6}-2}{\sqrt{6}-2}$

$= \dfrac{3\sqrt{12}+3\sqrt{6}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{3}\right)^2} - \dfrac{4\sqrt{18}+4\sqrt{6}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{2}\right)^2} + \dfrac{2\sqrt{18}-4\sqrt{3}}{\left(\sqrt{6}\right)^2 - (2)^2}$

$= \dfrac{6\sqrt{3}+3\sqrt{6}}{6-3} - \dfrac{12\sqrt{2}+4\sqrt{6}}{6-2} + \dfrac{6\sqrt{2}-4\sqrt{3}}{6-4}$

$= \dfrac{6\sqrt{3}+3\sqrt{6}}{3} - \dfrac{12\sqrt{2}+4\sqrt{6}}{4} + \dfrac{6\sqrt{2}-4\sqrt{3}}{2}$

$= \dfrac{24\sqrt{3}+12\sqrt{6}-36\sqrt{2}+36\sqrt{2}-24\sqrt{3}}{12} = \dfrac{0}{12} = 0.$

4. Solution:

$\dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \dfrac{1}{\sqrt{8}+3}$

$= \dfrac{1}{1+\sqrt{2}} \times \dfrac{1-\sqrt{2}}{1-\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} \times \dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$+ \dfrac{1}{\sqrt{3}+\sqrt{4}} \times \dfrac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}} + \ldots + \dfrac{1}{\sqrt{8}+3} \times \dfrac{\sqrt{8}-3}{\sqrt{8}-3}$

$= \dfrac{1-\sqrt{2}}{-1} + \dfrac{\sqrt{2}-\sqrt{3}}{-1} + \dfrac{\sqrt{3}-\sqrt{4}}{-1} + \ldots + \dfrac{\sqrt{8}-3}{-1}$

$= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} + \sqrt{4} - \ldots - 8\sqrt{3} + 3$

$= -1 + 3 = 2$

5. Solution:

Here, $\dfrac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{3}} + \dfrac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = a + \sqrt{15}b$

$\dfrac{\left(2\sqrt{5}+\sqrt{3}\right)^2 + \left(2\sqrt{5}-\sqrt{3}\right)^2}{\left(2\sqrt{5}-\sqrt{3}\right)\left(2\sqrt{5}+\sqrt{3}\right)} = a + \sqrt{15}b$

$\dfrac{20+3+4\sqrt{15}+20+3-4\sqrt{15}}{20-3} = a + \sqrt{15}b$

$\dfrac{46}{17} = a + \sqrt{15}b$

Comparing rational and irrational parts, we have

$a = \dfrac{46}{17}$ and $b = 0$

Assertion and Reason Answers-

1. (c) Assertion is correct statement but reason is wrong statement.

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Polynomials 2

Key Concepts

1. A **polynomial** p(x) in one variable x is an algebraic expression in x of the form $p(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_2x^2 + a_1x + a_0$, where.
   i. $a_0, a_1, a_2 \ldots a_n$ are constants
   ii. x is a variable
   iii. $a_0, a_1, a_2, \ldots\ldots a_n$ are respectively the **coefficients** of x^i
   iv. Each of $a_nx^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, \ldots\ldots a_2x_2, a_1x$, a, 0 with $a_n \neq 0$, is called a **term** of a polynomial.
2. The highest power of the variable in a polynomial is called the **degree** of the polynomial.
3. A polynomial with one term is called a **monomial**.
4. A polynomial with two terms is called a **binomial**.
5. A polynomial with three terms is called a **trinomial**.
6. A polynomial with degree zero is called a constant polynomial. For example: 1, –3. The degree of non-zero constant polynomial is zero.
7. A polynomial of degree one is called a **linear polynomial**. It is of the form ax + b. For example: x – 2, 4y + 89, 3x – z.
8. A polynomial of degree two is called a quadratic polynomial. It is of the form $ax^2 + bx + c$. where a, b, c are real numbers and $a \neq 0$ For example: $x^2 - 2x + 5$ etc.
9. A polynomial of degree three is called a cubic polynomial and has the general form $ax^3 + bx^2 + cx + d$. For example: $x^2 + 2x^2 - 2x + 5$ etc.
10. A **bi-quadratic polynomial** p(x) is a polynomial of degree four which can be reduced to quadratic polynomial in the variable $z = x^2$ by substitution.
11. The constant polynomial 0 is called the **zero polynomial**. Degree of zero polynomial is not defined.
12. The **value of a polynomial** f(x) at x = p is obtained by substituting x = p in the given polynomial and is denoted by f(p).
13. A real number 'a' is a **zero** or root of a polynomial p(x) if p (a) = 0.
14. The number of real zeroes of a polynomial is less than or equal to the degree of polynomial.
15. Finding a zero or root of a polynomial f(x) means solving the polynomial equation f(x) = 0.
16. A non-zero constant polynomial has no zero.
17. Every real number is a zero of a zero polynomial.
18. **Division algorithm**

    If p(x) and g(x) are the two polynomials such that degree of p(x) ≥ degree of g(x) and g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that:

    p(x) = g(x) q(x) + r(x)

    where, r(x) = 0 or degree of r(x) < degree of g(x).
19. **Remainder theorem**

    Let p(x) be any polynomial of degree greater than or equal to one and let a be any real number. If p(x) is divided by the linear polynomial (x – a), then remainder is p(a).

If the polynomial $p(x)$ is divided by $(x + a)$, the remainder is given by the value of $p(-a)$.

If $p(x)$ is divided by $ax + b = 0$; $a \neq 0$, the remainder is given by

$$P\left(\frac{|-b|}{a}\right); a \neq 0$$

If $p(x)$ is divided by $ax - b = 0$, $a \neq 0$, the remainder is given by

$$P\left(\frac{b}{a}\right); a \neq 0$$

20. **Factor theorem**

Let $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

21. **Converse of factor theorem**

Let $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number. If $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

i. $(x + a)$ is a factor of a polynomial $p(x)$ iff $p(-a) = 0$.
ii. $(ax - b)$ is a factor of a polynomial $p(x)$ iff $p(b/a) = 0$.
iii. $(ax + b)$ is a factor of a polynomial $p(x)$ iff $p(-b/a) = 0$.
iv. $(x - a)(x - b)$ is a factor of a polynomial $p(x)$ iff $p(a) = 0$ and $p(b) = 0$.

22. For applying factor theorem, the divisor should be either a linear polynomial of the form $(ax + b)$ or it should be reducible to a linear polynomial.

23. A quadratic polynomial $ax^2 + bx + c$ is **factorised by splitting the middle term** by writing b as $ps + qr$ such that $(ps)(qr) = ac$.

Then, $ax^2 + bx + c = (px + q)(rx + s)$

24. An **algebraic identity** is an algebraic equation which is true for all values of the variables occurring in it.

25. Some useful **quadratic identities:**

i. $(x + y)^2 = x^2 + 2xy + y^2$
ii. $(x - y)^2 = x^2 - 2xy + y^2$
iii. $(x - y)(x - y) = x^2 - y^2$
iv. $(x + a)(x + b) = x^2 + (a + b)x + ab$
v. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here x, y, z are variables and a, b are constants.

26. Some useful **cubic identities**:

i. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
ii. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
iii. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
iv. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
v. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
vi. if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

Here, x, y and z are variables.

Polynomial

Polynomials are expressions with one or more terms with a non-zero coefficient. A polynomial can have more than one term. In the polynomial, each expression in it is called a term. Suppose $x^2 + 5x + 2$ is polynomial, then the expressions x^2, $5x$ and 2 are the terms of the polynomial. Each term of the polynomial has a coefficient. For example, if $2x + 1$ is the polynomial, then the coefficient of x is 2.

The real numbers can also be expressed as polynomials. Like 3, 6, 7, are also polynomials without any variables. These are called constant polynomials. The constant polynomial 0 is called zero polynomial. The exponent of the polynomial should be a whole number. For example, $x^{-2} + 5x + 2$, cannot be considered as a polynomial, since the exponent of x is –2, which is not a whole number.

The highest power of the polynomial is called the degree of the polynomial. For example, in $x^3 + y^3 + 3xy(x + y)$, the degree of the polynomial is 3. For a non-zero constant polynomial, the degree is zero. Apart from these, there are other types of polynomials such as:

Polynomials in One Variable

The formulas with only one variable are known as polynomials in one variable. A polynomial is a mathematical statement made up of variables and coefficients that involves the operations of addition, subtraction, multiplication, and exponentiation.

Below are Some Instances of Polynomials in One Variable:

$x^2 + 3x - 2$

$3y^3 + 2y^2 - y + 1$

$m^4 - 5m^2 + 8m - 3$

Coefficient of Polynomials.

A coefficient is a number or quantity that is associated with a variable. It's generally an integer multiplied by the variable immediately adjacent to it.

For example, in the expression 3x, 3 is the coefficient but in the expression $x^2 + 3$, 1 is the coefficient of x^2.

Terms of Polynomial.

Polynomial terms are the portions of the equation that are usually separated by "+" or "–" marks. As a result, each term in a polynomial equation is a component of the polynomial. The number of terms in a polynomial like $2^2 + 5 + 4$ is 3.

Types of Polynomials:

Types of Polynomials	Meaning	Example
Zero or constant polynomial	A constant polynomial has its coefficients equal to 0. Whereas a zero polynomial is the additive identity of the additive groups of polynomials such as f(x) = 0. In a constant polynomial, the degree is 0 whereas in a zero polynomial, the degree is undefined or written as –1.	**3 or $3x^0$**
Linear polynomial	Linear polynomials are polynomials having a degree of 1 as the degree of the polynomial. The greatest exponent of the variable(s) in linear polynomials is 1.	**$x + y^4$** **5m + 7n** **2p**
Quadratic polynomial	Quadratic polynomials are polynomials having a degree of 2 as the degree of the polynomial.	**$8x^2 + 7y - 9$** **$m^2 + mn - 6$**
Cubic polynomial	Cubic polynomials are polynomials having a degree of the polynomial.	**$3x^3$** **$P^3 + pq + 7$**

Degree of Polynomial

The largest exponential power in a polynomial equation is called its degree. Only variables are taken into account when determining the degree of any polynomial; coefficients are ignored.

$4x^5 + 2x^3 - 20$

In the above polynomial degree will be 5.

Zeros of Polynomials

The polynomial zeros are the x values that fulfil the equation y = f(x). The zeros of the polynomial are the values of x for which the y value is equal to zero, and f(x) is a function of x. The degree of the equation y = f(x), determines the number of zeros in a polynomial.

Factorization of Polynomials

You know that any polynomial of the form p(a) can also be written as p(a) = g(a) × h(a) + R(a)

Dividend = Quotient × Divisor + Remainder

If the remainder is zero, then p(a) = g(a) x h(a). That is, the polynomial p(a) is a product of two other polynomials g (a) and h(a). For example, $3a + 6a^2 = 3a \times (1 + 2a)$.

A polynomial may be expressed in more than one way as the product of two or more polynomials.

Study the polynomial $3a + 6a^2 = 3a \times (1 + 2a)$.

This can also be factorized as $3a + 6a^2 = 6a \times \left(\frac{1}{2} + a\right)$.

Methods of Factorizing Polynomials

A polynomial can be factorised in a number of ways.

- Factorization, which is done by dividing the expression by the HCF of the words in the provided expression.
- Factorization by grouping the terms of the expression.
- Factorization using identities.

Factorization is achieved by dividing the expression by the HCF of the given expression's terms.

The biggest monomial in a polynomial is the HCF, which is a factor of each term in the polynomial. We can factorise a polynomial by determining the expression's Highest Common Factor (HCF) and then dividing each term by its HCF. The factors of the above equation are HCF and the quotient achieved.

Steps for Factorization

- Determine the HCF of the supplied expression's terms.
- Find the quotient by dividing each term of the provided equation by the HCF.
- As a product of HCF and quotient, write the given expression.

Factorization by Grouping the Expression's Terms

We come encounter polynomials in a variety of circumstances, and they may or may not contain common factors among their components. In such instances, we arrange the expression's terms so that common factors exist among the terms of the resulting groups.

Steps for Factorization by Grouping

- If required, rearrange the terms.
- Assemble the provided phrase into groups, each with its own common component.
- Determine each group's HCF.
- Find out what the other component is.
- Convert the phrase to a product of the common and additional factors.

Factorization Using Identities

To Locate the Products, Recall the Following Identities:

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$

4. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
5. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Observe that the LHS in the identities are all factors and the RHS are their products. Thus, we can write the factors as follows:

Factors of $a^2 - 2ab + b^2$ are $(a - b)$ and $(a - b)$ Factors of $a^2 + 2ab + b^2$ are $(a + b)$ and $(a + b)$ Factors of $a^2 - b^2$ are $(a + b)$ and $(a - b)$ Factors of $a^3 + 3a^2b + 3ab^2 + b^3$ are $(a + b)$, $(a + b)$ and $(a + b)$.

Factors of $a^3 - 3a^2b + 3ab^2 - b^3$ are $(a - b)$, $(a - b)$ and $(a - b)$ Factors of $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ are $(a + b + c)$ and $(a + b + c)$.

We may deduce from the preceding identities that a given statement in the form of an identity can be expressed in terms of its components.

Steps for Factorization Using Identities

Recognize the correct persona.

In the form of the identity, rewrite the provided statement.

Using the identity, write the factors of the given equation.

$a^3 \pm b^3 \pm 3ab(a \pm b) = (a \pm b)^3$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$x^3 \pm y^3 = (a \pm y) = (x^2 \pm xy \pm y^2)$

Factorization of Trinomials of the Form $x^2 + bx + c$

Trinomials are expressions with three terms. For example, $x^2 + 14x + 49$ is a trinomial. All trinomials cannot be factorised using a single approach. We must investigate the pattern in trinomials and select the best approach for factorising the given trinomial.

Factorizing a Trinomial by Splitting the Middle Term

The product of two binomials of the type $(x + a)$ and $(x + b)$ is $(x + a) \times (x + b)$ $x^2 + x(a + b) + ab$ [a trinomial]

CHAPTER : 2 POLYNOMIALS

Polynomials

Polynomial

An algebraic expression of the form:
$f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots a_1x^1 + a_0x^0$

Polynomial in one variable:
$ax^3 - bx^2 - cx + d$

Types

Terms

Polynomial	Example
Constant (or independent)	4, – 7/5
Zero	Degree not defined (constant polynomial 0)
Monomial	$4x$
Binomial	$2x + 3$
Trinomial	$3x^2 + 7x + 2$

Degree

Polynomial	Example	Degree
Linear	$3x + 2$	1
Quadratic	$2x^2 + 3x + 1$	2
Cubic	$7y^3 + 6y^2 + 2y + 2$	3

Algebraic Identities

$(x+y)^2$	$x^2 + 2xy + y^2$
$(x-y)^2$	$x^2 - 2xy + y^2$
$x^2 - y^2$	$(x - y)(x + y)$
$(x+a)(x+b)$	$x^2 + (a + b)x + ab$
$(x + y + z)^2$	$x^2+y^2+z^2+2xy+2yz+2zx$ $= x^2+y^2+z^2+2(xy+yz+zx)$
$(x + y)^3$	$x^3 + y^3 + 3xy(x + y)$
$(x - y)^3$	$x^3 - y^3 - 3xy(x - y)$
$x^3+y^3+z^3-3xyz$	$(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$
If $x + y + z = 0$	(i) $x^3+y^3+z^3 = 3xyz$ (ii) $\frac{x^2}{yz} + \frac{y^2}{xz} + \frac{z^2}{xy} = 3$
$x^3 + y^3$	$(x+y)(x^2-xy+y^2)$
$x^3 - y^3$	$(x-y)(x^2+xy+y^2)$

Theorems

Factor Theorem

if $p(x)$ is a polynomial of degree $n \geq 1$, a: any real number

(i) if $(x-a)$ is a factor of $p(x)$, then $p(a) = 0$

(ii) if $p(a) = 0$, then $(x-a)$ is a factor of $p(x)$.

Remainder Theorem

if $p(x)$ polynomial of degree $n>1$, is divided by $x-a$, $p(a)$ is the remainder

Divident = (Divisor × Quotient) + Remainder

Zeroes of polynomial

Number that satisfies the equation

Example

$P(x) = 2x + 1$
find zeroes of the polynomial

$P(x) = 0$
$2x + 1 = 0$
$x = -1/2$

$-1/2$ is the zeros of the polynomial

Important Questions

Multiple Choice Questions

1. $x^2 - 2x + 1$ is a polynomial in:
   (a) One Variable
   (b) Two Variables
   (c) Three variable
   (d) None of the above
2. The coefficient of x^2 in $3x^3+2x^2-x+1$ is:
   (a) 1
   (b) 2
   (c) 3
   (d) −1
3. A binomial of degree 20 in the following is:
   (a) 20x + 1
   (b) x/20 + 1
   (c) x20 + 1
   (d) x2 + 20
4. The degree of $4x^3 - 12x^2 + 3x + 9$ is
   (a) 0
   (b) 1
   (c) 2
   (d) 3
5. $x^2 - x$ is _______ polynomial.
   (a) Linear
   (b) Quadratic
   (c) Cubic
   (d) None of the above
6. $x - x^3$ is a _______ polynomial.
   (a) Linear
   (b) Quadratic
   (c) Cubic
   (d) None of the above
7. $1 + 3x$ is a ________ polynomial.
   (a) Linear
   (b) Quadratic
   (c) Cubic
   (d) None of the above
8. The value of $f(x) = 5x-4x^2+3$ when $x = -1$, is:
   (a) 3
   (b) −12
   (c) −6
   (d) 6
9. The value of $p(t) = 2 + t + 2t^2 - t^3$ when $t = 0$ is
   (a) 2
   (b) 1
   (c) 4
   (d) 0
10. The zero of the polynomial $f(x) = 2x + 7$ is
   (a) 2/7
   (b) −2/7
   (c) 7/2
   (d) −7/2

Very Short Questions:

1. Factorise: $125x^3 - 64y^3$
2. Find the value of $(x + y)^2 + (x - y)^2$.
3. If $p(x)= x^2 - 2\sqrt{2}x + 1$, then find the value of $p\left(2\sqrt{2}\right)$.
4. Find the value of m, if x + 4 is a factor of the polynomial $x^2 + 3x + m$.
5. Find the remainder when $x^3 + x^2 + x + 1$ is divided by $x-\frac{1}{2}$ using remainder theorem.
6. Find the common factor in the quadratic polynomials $x^2 + 8x + 15$ and $x^2 + 3x - 10$.

Short Questions:

1. Expand:
   i) $\left(y-\sqrt{3}\right)^2$
   ii) $(x - 2y - 3z)^2$
2. If $1+\frac{1}{x}=7$ then find the value of $x^3+\frac{1}{x^3}$.
3. Show that p−1 is a factor of $p^{10}+p^8+p^6-p^4-p^2-1$.
4. If $3x + 2y = 12$ and $xy = 6$, find the value of $27x^3+8y^3$.
5. Factorise: $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$.
6. Factorise: $1 - 2ab - (a^2 + b^2)$.
7. Factories: $27a^3+\frac{1}{64b^3}+\frac{27a^2}{4b}+\frac{9a}{16b^2}$.

Long Questions:

1. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

2. Factorise: $(m + 2n)^2 x^2 - 22x (m + 2n) + 72$.
3. If $x - 3$ is a factor of $x^2 - 6x + 12$, then find the value of k. Also, find the other factor of the – polynomial for this value of k.
4. Find a and b so that the polynomial x^3-10x^2+ax+b is exactly divisible by the polynomials $(x - 1)$ and $(x - 2)$.
5. Factorise: $x^2 - 6x^2 + 11x - 6$.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: If $f(x) = 3x^7-4x^6+x+9$ is a polynomial, then its degree is 7.

   Reason: Aromatic aldehydes are almost as reactive as formaldehyde.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The expression $3x^4 - 4x^{3/2} + x^2 = 2$ is not a polynomial because the term $- 4x^{3/2}$ contains a rational power of x.

   Reason: The highest exponent in various terms of an algebraic expression in one variable is called its degree.

Answer Key

Multiple Choice Questions

1. (a) One Variable
2. (b) 2
3. (c) $x^{20} + 1$
4. (d) 3
5. (b) Quadratic
6. (c) Cubic
7. (a) Linear
8. (c) –6
9. (a) 2
10. (d) –7/2

Very Short Answer:

1. $125x^3 - 6443 = (5x)^3 - (4y)^3$
   By using $a^3 - b^3 = (a-b)(a^2+a^b+b^2)$, we obtain
   $125x^3-64y^3 = (5x - 4y)(25x^2 + 20xy + 16y^2)$
2. $(x + y)^2 + (x - y)^2 = x^2 + y^2 + 2xy + x^2 + y^2 - 2xy$
   $= 2x^2 + 2y^2 = 21x^2 + y^2$
3. Put $x = 2\sqrt{2}$ in p(x), we obtain
   $p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2}) + 1$
   $= (2\sqrt{2})^2 - (2\sqrt{2})^2 + 1 = 1$
4. Let $p(x) = x^2 + 3x + m$
   Since $(x + 4)$ or $(x - (-4))$ is a factor of p(x).
   $\therefore p(-4) = 0$
   $\Rightarrow (-4)^2 + 3(-4) + m = 0$
   $\Rightarrow 16 - 12 + m = 0$
   $\Rightarrow m = -4$
5. Let $p(x) = x^3 + x^2 + x + 1$ and $q(x) = x - \frac{1}{2}$
   Here, p(x) is divided by q(x)
   ∴ By using remainder theorem, we have
   $$\text{Remainder} = p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1$$
   $$= \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1 = \frac{1+2+4+8}{8} = \frac{15}{8}.$$

6. $x^2 + 8x + 15 = x^2 + 5x + 3x + 15 = (x + 3)(x + 5)$

$x^2 + 3x - 10 = x^2 + 5x - 2x - 10 = (x - 2)(x + 5)$

Clearly, the common factor is $x + 5$.

Short Answer:

1. **Solution:**

$(y - \sqrt{3})^2 = y^2 - 2 \times y \times \sqrt{3} + (\sqrt{3})^2$

$= y^2 - 2\sqrt{3}y + 3$ $(x - 2y - 3z)^2$

$= x^2 + 1 - (2y)^2 + (-3z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-3z) + 2 \times (-3z) \times x$

$= x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx$

2. **Solution:**

We have $x + \frac{1}{x} = 7$

Cubing both sides, we have

$$\left(x + \frac{1}{x}\right)^3 = 7^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 7 = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 343 - 21 = 322$$

3. **Solution:**

Let $f(p) = p^{10} + p^8 + p^6 - p^4 - p^2 - 1$

Put $p = 1$, we obtain

$f(1) = 1^{10} + 1^8 + 1^6 - 1^4 - 1^2 - 1$

$= 1 + 1 + 1 - 1 - 1 - 1 = 0$

Hence, $p - 1$ is a factor of $p^{10} + p^8 + p^6 - p^4 - p^2 - 1$

4. **Solution:**

We have $3x + 2y = 12$

On cubing both sides, we have

$\Rightarrow (3x + 2y)^3 = 12^3$

$\Rightarrow (3x)^3 + (2y)^3 + 3 \times 3x \times 2y(3x + 2y) = \sqrt{278}$

$\Rightarrow 27x^3 + 8y^3 + 18xy(3x + 2y) = \sqrt{278}$

$\Rightarrow 27x^3 + 8y^3 + 18 \times 6 \times 12 = \sqrt{278}$

$\Rightarrow 27x^3 + 8y^3 + 1296 = \sqrt{278}$

$\Rightarrow 27x^3 + 8y^3 = \sqrt{278} - 1296$

$\Rightarrow 27x^3 + 8y^3 = 432$

5. **Solution:**

$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-2z) + 2(-2z)(2x)$

By using $a^2 + b^2 + 2ab + 2bc + 2ca = (a + b + c)^2$, we obtain

$= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$

6. **Solution:**

$1 - 2ab - (a^2 + b^2) = 1 - (a^2 + b^2 + 2ab)$

$= 12 - (a + b)^2$

$= (1 + a + b)(1 - a - b)$

$[\because x^2 - y^2 = (x + y)(x - y)]$

7. **Solution:**

$$27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}$$

$$= (3a)^3 + \frac{1}{(4b)^3} + 3.(3a).\left(\frac{1}{4b}\right)\left(3a + \frac{1}{4b}\right)$$

By using $x^3 + y^3 + 3xy(x+y) = (x+y)^3$, we have

$$= \left(3a + \frac{1}{4b}\right)^3$$

Long Answer:

1. **Solution:**

L.H.S. $= (a + b + c)^3 - a^3 - b^3 - c^3$

$= \{(a + b + c)^3 - 3\} - \{b^3 + c^3\}$

$= (a + b + c - a)\{(a + b + c)^2 + a^2 + a(a + b + c)\} - (b + c)(b^2 + c^2 - bc)$

$= (b + c)\{a^2 + b^2 + 2ab + 2bc + 2ca + a^2 + a^2 + ab + ac - b^2 - a^2 + bc)$

$= (b + c)(3a^2 + 3ab + 3bc + 3ca\}$

$= 3(b + c)\{a^2 + ab + bc + ca\}$

$= 3(b + c)\{\{a^2 + ca) + (ab + bc)\}$

$= 3(b + c)\{a(a + c) + b(a + c)\}$

$= 3(b + c)(a + c)(a + b)$

$= 3(a + b)(b + c)(c + a) =$ R.H.S.

2. **Solution:**

Let $m + 2n = a$

$\therefore (m + 2n)^2 x^2 - 22x(m + 2n) + 72 = a^2x^2 - 22ax + 72$

$= a^2x^2 - 18ax - 4ax + 72$

$= ax(ax - 18) - 4(ax - 18)$

$= (ax - 4)(ax - 18)$

$= \{(m + 2n)x - 4)\}\{(m + 2n)x - 18)\}$

$= (mx + 2nx - 4)(mx + 2nx - 18)$.

3. **Solution:**

Here, $x - 3$ is a factor of $x^2 - kx + 12$

$\therefore$ By factor theorem, putting $x = 3$, we have remainder 0.

$\Rightarrow (3)^2 - k(3) + 12 = 0$

$\Rightarrow 9 - 3k + 12 = 0$

$\Rightarrow 3k = 21$

$\Rightarrow k = 7$

Now, $x^2 - 7x + 12 = x^2 - 3x - 4x + 12$

$= x(x - 3) - 4(x - 3)$

$= (x - 3)(x - 4)$

Hence, the value of k is 7 and other factor is $x - 4$.

4. **Solution:**

Let $p(x) = x^3 - 10x^2 + ax + b$

Since p(x) is exactly divisible by the polynomials $(x - 1)$ and $(x - 2)$.

$\therefore$ By putting x = 1, we obtain

$(1)^3 - 10(1)^2 + a(1) + b = 0$

$\Rightarrow a + b = 9$

And by putting x = 2, we obtain

$(2)^3 - 10(2)^2 + a(2) + b = 0$

$8 - 40 + 2a + b = 0$

$\Rightarrow 2a + b = 32$

Subtracting (i) from (ii), we have

$a = 23$

From (i), we have $23 + b = 9 = b = -14$

Hence, the values of a and b are a=23 and b=−14

5. **Solution:**

Let $p(x) = x^2 - 6x^2 + 11x - 6$

Here, constant term of p(x) is −6 and factors of −6 are ± 1, ± 2, ± 3 and ± 6

By putting x = 1, we have

$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

$\therefore (x - 1)$ is a factor of p(x)

By putting x = 2, we have

$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

$\therefore (x - 2)$ is a factor of p(x)

By putting x = 3, we have

$p(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

$\therefore (x - 3)$ is a factor of p(x). Since p(x) is a polynomial of degree 3, so it cannot have more than three linear factors.

$\therefore x^3 - 6x^2 + 11x - 6 = k(x - 1)(x - 2)(x - 3)$

By putting x = 0, we obtain

$0 - 0 + 0 - 6 = k(-1)(-2)(3)$

$-6 = -6k$

$k = 1$

Hence, $x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$.

Assertion and Reason Answers-

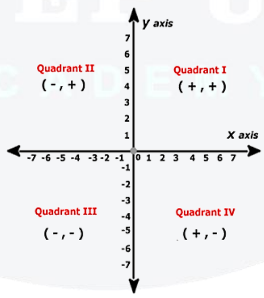
1. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. (b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

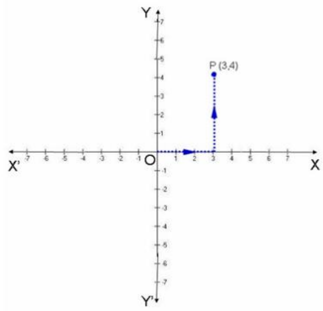
Coordinate Geometry 3

Key Concepts

1. Two perpendicular number lines intersecting at point zero are called **coordinate axes**. The horizontal number line is the **x-axis** (denoted by X'OX) and the vertical one is the **y-axis** (denoted by Y'OY). The point of intersection of x-axis and y-axis is called **origin** and denoted by 'O'.
2. **Cartesian plane** is a plane obtained by putting the coordinate axes perpendicular to each other in the plane. It is also called coordinate plane or xy plane.
3. The **x-coordinate** of a point is its perpendicular distance from y-axis.
4. The **y-coordinate** of a point is its perpendicular distance from x-axis.
5. The point where the x axis and the y axis intersect is represented by coordinate points (0, 0) and is called the **origin**.
6. The **abscissa** of a point is the x-coordinate of the point. The **ordinate** of a point is the y-coordinate of the point.
7. If the abscissa of a point is x and the ordinate of the point is y, then (x, y) are called the **coordinates** of the point.
8. The axes divide the Cartesian plane into four parts called the **quadrants** (one fourth part), numbered I, II, III and IV anticlockwise from OX.
9. Sign of coordinates depicts the quadrant in which it lies. The coordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant.



10. The coordinates of a point on the x-axis are of the form (x, 0) and that of the point on y-axis are (0, y).
11. To plot a point P (3, 4) in the Cartesian plane, start from origin and count 3 units on the positive x axis then move 4 units towards positive y axis. The point at which we will arrive will be the point P (3, 4).



12. If x ≠ y, then (x, y) ≠ (y, x) and if (x, y) = (y, x), then x = y.

Cartesian System

Cartesian plane & Coordinate Axes

Cartesian Plane: A cartesian plane is defined by two perpendicular number lines, A horizontal line(x–axis) and a vertical line (y–axis).

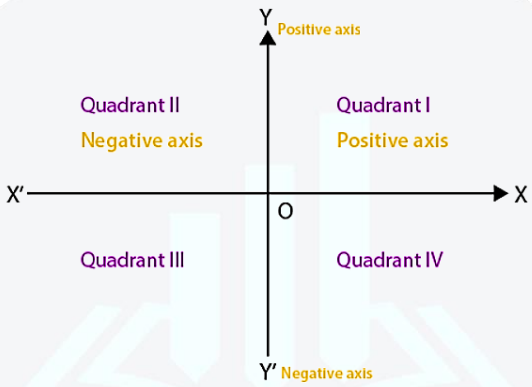
These lines are called coordinate axes. The Cartesian plane extends infinitely in all directions.

Origin: The coordinate axes intersect each other at right angles, The point of intersection of these two axes is called Origin.

Co-ordinate system is used to locate the position of a point in a plane using two perpendicular lines. Points are represented in the form of coordinates (x, y) in two-dimension with respect to x- and y- axes. In this article, we will learn about Cartesian Coordinate system.

To understand the need of coordinate system, let us consider an example, suppose Rina is a girl in your class and she sits on the 3rd column and 5th row. Then, this position can be represented as (3, 5).

Two axes – vertical axis and perpendicular axis are reference lines of a rectangular system from which distances are measured. They are obtained as follows:



Explanation:

Take two number lines XX' and YY'. Place XX' in horizontal and write the numbers on it as we write in the number line. Similarly, place YY' in vertical and proceed writing numbers on it as we write in a number line. Combine both the lines in such a way that the two lines cross each other at their zeroes or origins. The horizontal line XX' is called the x-axis and the vertical line YY' is called the y-axis. The point where XX' and YY' cross is called the origin, and is denoted by O. Since the positive numbers lie on the directions OX and OY, OX and OY are called the positive directions of the x-axis and the y-axis respectively. Similarly, OX' and OY' are called the negative directions of the x- and y-axes respectively.

Important Terms:

Quadrants:

Moreover, the axes divide the plane into four parts and these four parts are called quadrants (one-fourth part). Thus, we have four quadrants numbered I, II, III and IV anticlockwise from OX.

Polynomial terms are the portions of the equation that are usually separated by "+" or "–" marks. As a result, each term in a polynomial equation is a component of the polynomial. The number of terms in a polynomial like $2^2 + 5 + 4$ is 3.

Points in different Quadrants.

Signs of coordinates of points in different quadrants:

I Quadrant: '+' x – coordinate and '+' y – coordinate. E.g. (2, 3)

II Quadrant: '–' x – coordinate and '+' y – coordinate. E.g. (–1, 4)

III Quadrant: '–' x – coordinate and '–' y – coordinate. E.g. (–3, –5)

IV Quadrant: '+' x – coordinate and '–' y – coordinate. E.g. (6, –1)

Cartesian Plane:

A plane consists of axes and quadrants. Thus, we call the plane the Cartesian Plane, or the Coordinate Plane, or the x-y plane. The axes are called the coordinate axes.

Cartesian coordinate system for one dimensional:

The Cartesian coordinate system for one dimensional space consists of a line. We choose a point O, origin on the line, a unit of length and orientation for the line. The orientation chooses which of the two half lines determined by O is the positive, and which is negative. Each point P of the line can be specified by its distance from O, taken with a negative or positive sign.

Number line:

A line with a chosen Cartesian system is called a number line. Every real number has a unique location on the line. Every point on the number line can be interpreted as a number.

Important Note:

The above depicts a two-dimensional system. In case of a three-dimensional system, we have three mutually perpendicular axes, namely x, y and z. It can be generalized to create n coordinates for any point in n-dimensional Euclidean space.

Abscissa and Ordinate

The x-coordinate of a point is its perpendicular distance from the y-axis measured along the x-axis and it is known as Abscissa.

The y-coordinate of a point is its perpendicular distance from the x-axis measured along the y-axis and it is known as Ordinate.

In writing the coordinates of a point in the coordinate plane, the x-coordinate comes first and then the y-coordinate. We place the coordinates in brackets as (x, y). The coordinates describe a point in the plane uniquely. It implies $(3,1) \neq (1,3)$ or in general $(x, y) \neq (y, x)$.

Consider an example point (5,6). Here abscissa = 5 and ordinate = 6.

Different Types of Coordinate Systems

We have mainly two types of coordinate systems as listed below:

Cartesian coordinate system

As stated above, it uses the concept of mutually perpendicular lines to denote the coordinate of a point. To locate the position of a point in a plane using two perpendicular lines, we use the cartesian coordinate system. Points are represented in the form of coordinates (x, y) in two-dimension with respect to x- and y- axes.

The x-coordinate of a point is its perpendicular distance from the y-axis measured along the x-axis and it is known as Abscissa. The y-coordinate of a point is its perpendicular distance from the x-axis measured along the y-axis and it is known as Ordinate.

Polar Coordinate System

Here, a point is chosen as the pole and a ray from this point is taken as the polar axis. Basically, we have two parameters namely angle and radius. The angle θ with the polar axis has a single line through the pole measured anti-clockwise from the axis to the line.

The point will have a unique distance from the origin (r). Thus, a point in Polar coordinate system is represented as a pair of coordinates (r, θ). The pole is represented by $(0, \theta)$ for any value of θ, where $r = 0$.

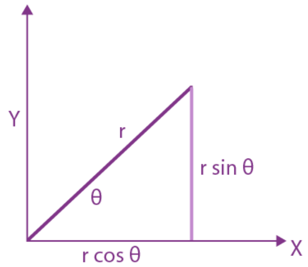
(r, θ), $(r, \theta + 2\pi)$ and $(-r, \theta + \pi)$ are all polar coordinates for the same point.

The distance from the pole is called the radial coordinate, radial distance or simply radius and the angular coordinate, polar angle or azimuth.

Consider the figure below that depicts the relationship between polar and cartesian coordinates.

$X = r \cos \theta$ and $y = r \sin \theta$

$r = (x^2 + y^2)$ ½ and $\tan \theta = (y/x)$



Polar equation of a curve consists of points of the form (r, θ).

In case of circle, the general equation for a circle with centre at (R, β) and radius a is $r^2 - 2rR\cos(\theta - \beta) + R^2 = a^2$.

Radial lines (those running through the pole) are represented by the equation: Θ = β.

Cartesian Formulae for the Plane

Distance between two points

The distance between two points of the plane (x1, y1) and (x2, y2) is given by

$$d = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]^{1/2}$$

In case of three-dimensional system, the distance formula between the points (x1, y1, z1) and (x2, y2, z2) is

$$d = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{1/2}$$

Representation of a vector

In two-dimensions, the vector from the origin to the point with the cartesian coordinates (x, y) can be written as r = xi + yj where i = (1,0) and j = (0,1) are unit vectors in the direction of the x-axis and y-axis respectively.

In case of three-dimensions, we will have r = xi + yj + zk, where k = (0,0,1) is the unit vector in the direction of z-axis.

Three Dimensional Geometry

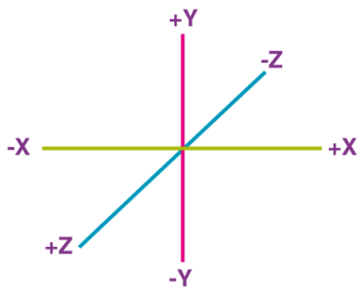
3D geometry involves the mathematics of shapes in 3D space and involving 3 coordinates which are x-coordinate, y-coordinate and z-coordinate. In a 3d space, three parameters are required to find the exact location of a point. For JEE, three-dimensional geometry plays a major role as a lot of questions are included in the exam. Here, the basic concepts of geometry involving 3-dimensional coordinates are covered which will help to understand different operations on a point in 3d plane.

Coordinate System in 3D Geometry

In 3 dimensional geometry, a coordinate system refers to the process of identifying the position or location of a point in the coordinate plane. To understand more about coordinate planes and system, refer to the coordinate geometry lesson which covers all the basic concepts, theorems, and formulas related to coordinate or analytic geometry.

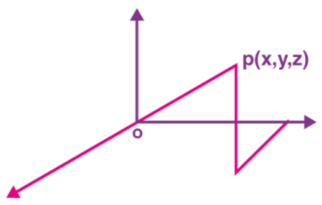
Rectangular coordinate system

Three lines perpendicular to each other pass through a common point. That common point is called the origin, the 3 lines the axes. They are x-axis, y-axis, z-axis respectively. O is the observer with respect to his position of any other point is measured. The position or coordinates of any point in 3D space is measured by how much he has moved along x, y and z-axis respectively. So if a point has a position (3, –4, 5) means he has moved 3 unit along positive x-axis, 4 unit along negative y-axis, 5 unit along positive z-axis.



Rectangular coordinate system – 3D Geometry

Distance from the Origin

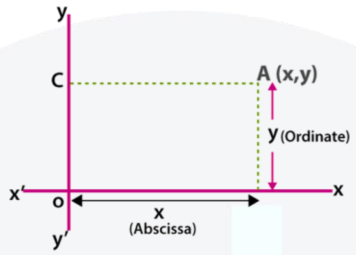


Distance from the Origin in 3D Space – 3D Geometry

Plotting on a Graph

Representation of a point on the Cartesian plane

Using the co-ordinate axes, we can describe any point in the plane using an ordered pair of numbers. A point A is represented by an ordered pair (x, y) where x is the abscissa and y is the ordinate of the point.

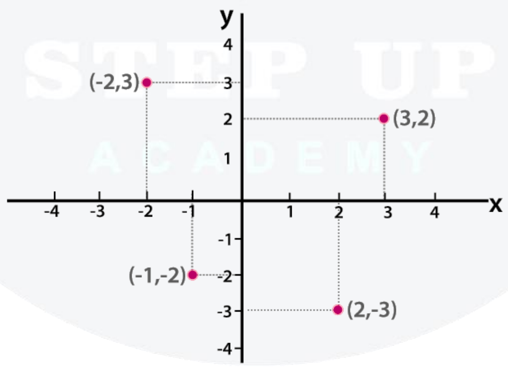


Position of a point in a plane

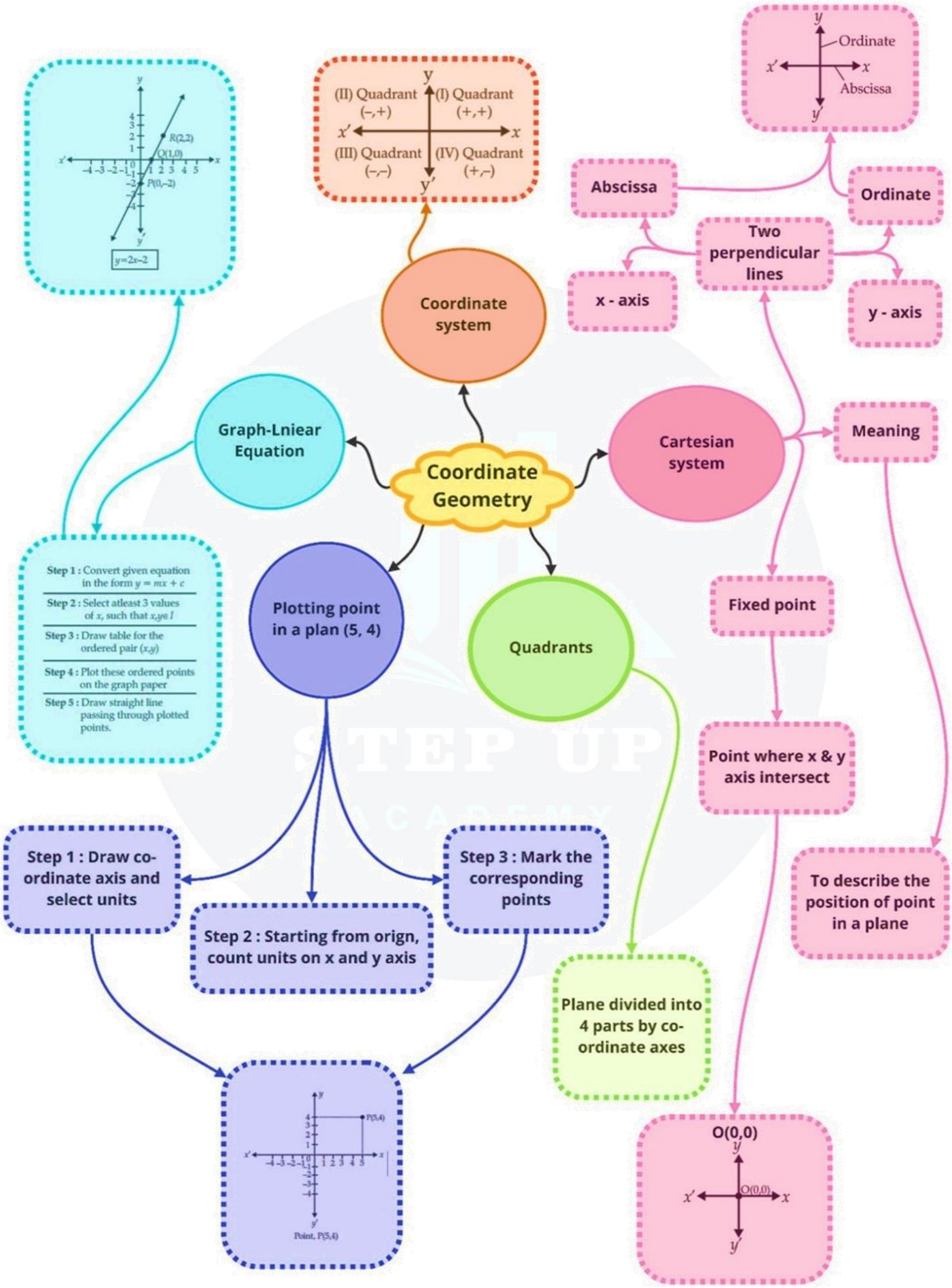
Plotting a point

The coordinate points will define the location in the cartesian plane. The first point (x) in the coordinates represents the horizontal axis, and the second point in the coordinates (y) represents the vertical axis.

Consider an example, Point (3,2) is 3 units away from the positive y-axis and 2 units away from the positive x-axis. Therefore, point (3, 2) can be plotted, as shown below. Similarly, (−2, 3), (−1, −2) and (2, −3) are plotted.



Class : 9th mathematics
Chapter- 3: Coordinate Geometry



Important Questions

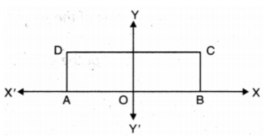
Multiple Choice Questions

1. If the coordinates of a point are (0, −4), then it lies in:
   (a) X-axis
   (b) Y-axis
   (c) At origin
   (d) Between x-axis and y-axis
2. If the coordinates of a point are (3, 0), then it lies in:
   (a) X-axis
   (b) Y-axis
   (c) At origin
   (d) Between x-axis and y-axis
3. If the coordinates of a point are (−3, 4), then it lies in:
   (a) First quadrant
   (b) Second quadrant
   (c) Third quadrant
   (d) Fourth quadrant
4. If the coordinates of a point are (−3, −4), then it lies in:
   (a) First quadrant
   (b) Second quadrant
   (c) Third quadrant
   (d) Fourth quadrant
5. The name of horizontal line in the cartesian plane which determines the position of a point is called:
   (a) Origin
   (b) X-axis
   (c) Y-axis
   (d) Quadrants
6. The name of vertical line in the cartesian plane which determines the position of a point is called:
   (a) Origin
   (b) X-axis
   (c) Y-axis
   (d) Quadrants
7. The section formed by horizontal and vertical lines determining the position of point in a cartesian plane is called:
   (a) Origin
   (b) X-axis
   (c) Y-axis
   (d) Quadrants
8. The point of intersection of horizontal and vertical lines determining the position of point in a cartesian plane is called:
   (a) Origin
   (b) X-axis
   (c) Y-axis
   (d) Quadrants
9. Points (1, 2), (−2, −3), (2, −3);
   (a) First quadrant
   (b) Do not lie in the same quadrant
   (c) Third quadrant
   (d) Fourth quadrant
10. If x coordinate of a point is zero, then the point lies on:
   (a) First quadrant
   (b) Second quadrant
   (c) X-axis
   (d) Y-axis

Very Short Questions:

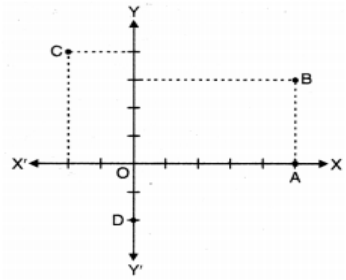
1. Write the signs convention of the coordinates of a point in the second quadrant.
2. Write the value of ordinate of all the points lie on x-axis.
3. Write the value of abscissa of all the points lie on y-axis.
4. If in coordinates of a point B(3, −2), signs of both coordinates are interchanged, then it will lie in which quadrant?
5. Find distances of points C(−3, −2) and D(5, 2) from x-axis and y-axis.
6. Find the values of x and y, if two ordered pairs (x – 3, – 6) and (4, x + y) are equal.
7. In which quadrant does the point (−1, 2) lie?
8. Find the distance of the point (0, −5) from the origin.
9. Write the shape of the quadrilateral formed by joining (1, 1), (6, 1), (4, 5) and (3, 5) on graph paper.

Short Questions:



1. In the given figure, ABCD is a rectangle with length 6 cm and breadth 3 cm. O is the mid-point of AB. Find the coordinates of A, B, C and D.

2. Write the coordinates of A, B, C and D from the figure given alongside.



3. A point lies on x-axis at a distance of 9 units from y-axis. What are its coordinates? What will be the coordinates of a point, if it lies on y-axis at a distance of −9 units from x-axis?
4. Plot the point P(2, −6) on a graph paper and from it draw PM and PN perpendiculars to x-axis and y-axis respectively. Write the coordinates of the points M and N.
5. Without plotting the points indicate the quadrant in which they lie, if :
   (i) ordinate is 5 and abscissa is −3
   (ii) abscissa is −5 and ordinate is −3
   (iii) abscissa is −5 and ordinate is 3
   (iv) ordinate is 5 and abscissa is 3
6. Plot the points A(1, 4), B(−2, 1) and C(4, 1). Name the figure so obtained on joining them in order and also, find its area.
7. Plot the following points, join them in order and identify the figure thus formed: A(1, 3), B(1, −1), C(7, −1) and D(7, 3).

Long Questions:

1. Plot the points A(3, 2), B(−2, 2), C(−2, −2) and D(3, −2) in the cartesian plane. Join these points and name the figure so formed.
2. Write the coordinates of two points on X-axis and two points on Y-axis which are at equal distances from the origin. Connect all these points and make them as vertices of quadrilateral. Name the quadrilateral thus formed.
3. On environment day, class-9 students got five plants of mango, silver oak, orange, banyan and amla from soil department. Students planted the plants and noted their locations as (x, y).

	Mango	Silver Oak	Orange	Banyan	Amla
x	2	3	0	−2	−2
y	0	4	7	4	0

Plot the points (x, y) in the graph and join them in the given order. Name the figure you get. Which social act is being done by students of class-9?

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The points (−3, 5) and (5, −3) are at different positions in the coordinate plane.

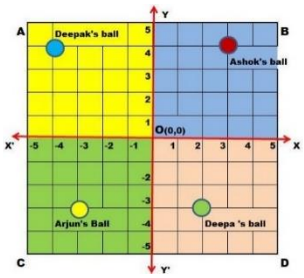
   Reason: If x ≠ y , then (x, y) ≠ (y,x)
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The point (−5 , 0) lies on y – axis and (0, −4) on x -axis.

   Reason: Every point on the x -axis has zero distance from x -axis and every point on the y -axis has zero distance from y -axis.

Case Study Questions:

1. Read the Source/ Text given below and answer these questions:



There is a square park ABCD in the middle of Saket colony in Delhi. Four children Deepak, Ashok, Arjun and Deepa went to play with their balls. The colour of the ball of Ashok, Deepak, Arjun and Deepa are red, blue, yellow and green respectively. All four children roll their ball from centre point O in the direction of XOY, X'OY, X'OY' and XOY'. Their balls stopped as shown in the above image.

Answer the following questions:

i. What are the coordinates of the ball of Ashok?
   a. (4, 3)
   b. (3, 4)
   c. (4, 4)
   d. (3, 3)

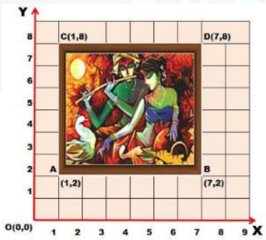
ii. What are the coordinates of the ball of Deepa?
   a. (2, –3)
   b. (3, 2)
   c. (2, 3)
   d. (2, 2)

iii. What the line XOX' is called?
   a. y-axis.
   b. ordinate.
   c. x-axis.
   d. origin.

iv. What the point O(0, 0) is called?
   a. y-axis.
   b. ordinate.
   c. x-axis.
   d. origin.

v. What is the ordinate of the ball of Arjun?
   a. –3
   b. 3
   c. 4
   d. 2

2. Read the Source/ Text given below and answer any four questions:



Rohit was putting up one of his paintings in his living room. Before this Rohit had put a grid on the wall where each unit measured equal to a foot. The upper-left corner of the frame is at point C(1, 8) and the upper-right corner at D(7, 8). The bottom-left corner is at A(1, 2) and the bottom-right corner at B(7, 2).

Please answer the following questions:

i. What is the width of the painting plus frame?
   a. 5 feet
   b. 8 feet
   c. 9 feet
   d. 6 feet

ii. What is the length of the painting plus frame?
   a. 9 feet
   b. 8 feet
   c. 6 feet
   d. 5 feet

iii. Which sides of the painting are parallel to x-axis?
   a. AB and CD
   b. AC and BD
   c. Diagonals AD and BC
   d. No one

iv. Which sides of the painting are parallel to y-axis?
   a. AB and CD
   b. AC and BD
   c. Diagonals AC and BD
   d. No one

v. Point A, B, C and D lie in which quadrant?
   a. I
   b. II
   c. III
   d. IV

Answer Key

Multiple Choice Questions

1. (b) Y-axis
2. (a) Y-axis
3. (b) Second quadrant
4. (c) Third quadrant
5. (b) X-axis
6. (c) Y-axis
7. (d) Quadrants
8. (a) Origin
9. (b) Do not lie in the same quadrant
10. (d) Y-axis

Very Short Answer:

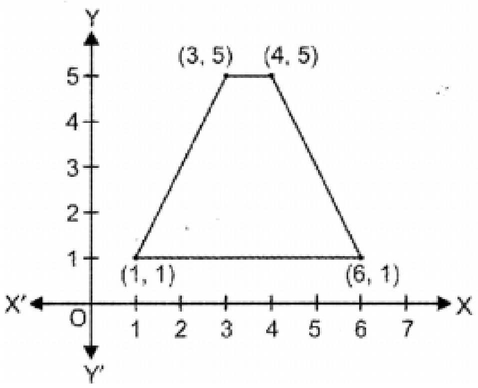
1. (–ve, +ve)
2. 0
3. 0
4. When signs of both coordinates of B(3, –2) are interchanged, then coordinates of new point are B'(–3, 2) and it will lie in second quadrant.
5. Distances of point C(–3, –2) from x-axis is 2 units in the negative direction and from y-axis is 3 units in the negative direction. Distances of point D(5, 2) from x-axis is 2 units and from y-axis is 5 units.
6. Here, two ordered pairs are equal.

   ⇒ Their first components are equal, and their second components are separately equal.

   $\Rightarrow x - 3 = 4$ and $x + y = -6$

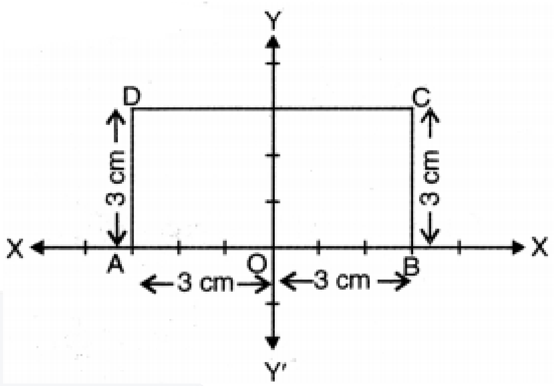
   $\Rightarrow x = 7$ and $7 + y = -6 \Rightarrow y = -13$

   Hence, $x = 7$ and $y = -13$.
7. (–1, 2) lie in second quadrant.
8. 5 units.
9. Trapezium.



Short Answer:

1. **Solution:**



We have taken 1cm = 1 unit and origin O is the mid-point of AB

∴ OA = OB = 3cm

and BC = AD = 3cm

Thus, the coordinates of A are (–3, 0)

the coordinates of B are (3, 0)

the coordinates of C are (3, 3)

the coordinates of D are (-3, 3)

2. **Solution:**

Coordinates of the point A are (5, 0)

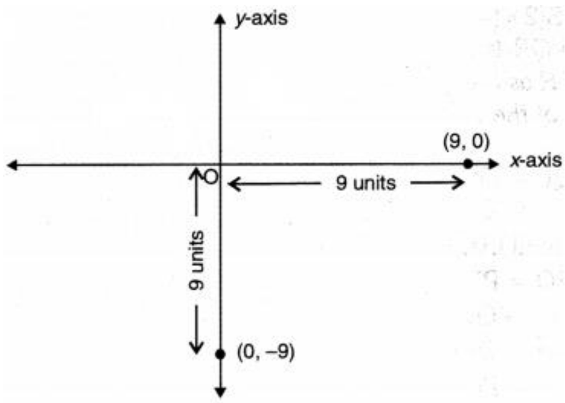
Coordinates of the point B are (5, 3)

Coordinates of the point C are (–2, 4)

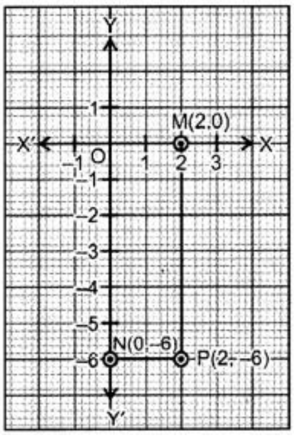
Coordinates of the point D are (0, –2)

3. **Solution:**

As shown in graph, the coordinates of a point which lies on x-axis at a distance of 9 units from y-axis are (9, 0) and the coordinates of a point which lies at a distance of -9 units from x-axis are (0, –9).



4. **Solution:**



5. **Solution:**

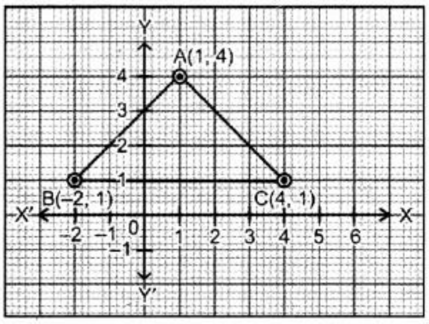
(i) Clearly, point (–3, 5) lies in 2nd quadrant.

(ii) Clearly, point (–5, –3) lies in 3rd quadrant.

(ii) Clearly, point (–5, 3) lies in 2nd quadrant.

(iv) Clearly, point (3, 5) lies in 1st quadrant.

6. **Solution:**



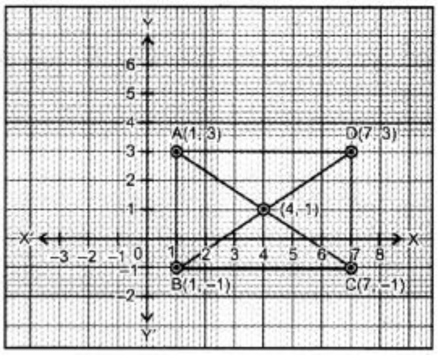
Triangle

Area of ΔABC = $\frac{1}{2}$ × BC × Height

= $\frac{1}{2}$ × 6 × 3

= 9 sq. units

7. **Solution:**



ABCD is a rectangle.

Point of intersection of the diagonals AC and BD is (4, 1)

Long Answer:

1. **Solution:**

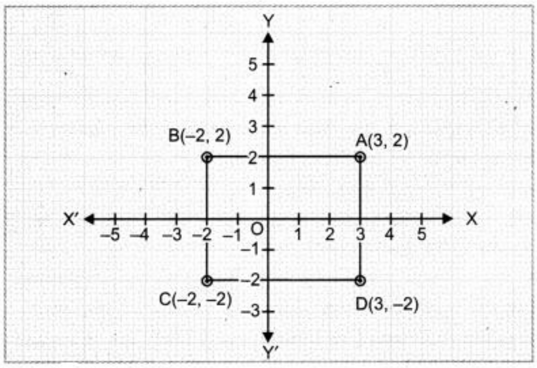
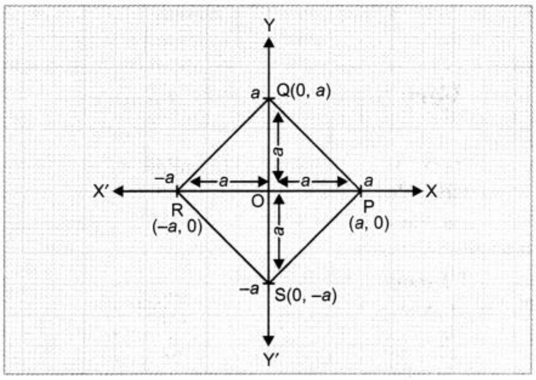


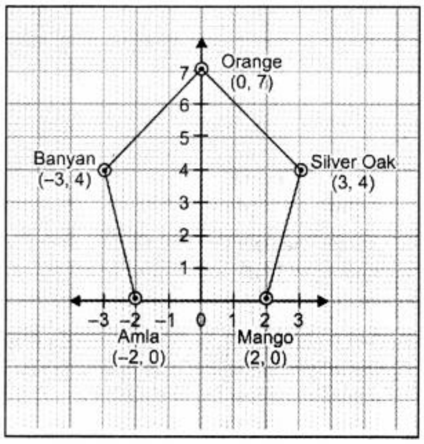
Figure so formed is ABCD a rectangle.

2. **Solution:**

Let a be the equal distance from origin on both axes. Now, the coordinates of two points on equal distance ‘a ‘on x-axis are P(a, 0) and R(–a, 0). Also, the coordinates of two points on equal distance ‘a’ on Y-axis are Q(0, a) and S(0, –a). Join all the four points on the graph. Now, PQRS, thus formed is a square.



3. **Solution:**



The given trees (points) are Mango (2, 0), Silver Oak (3, 4), Orange (0,7), Banyan (–3, 4) and Amla (–2, 0). The location of these trees are Orange (0,7) shown in the graph.

On joining the points of mango, silver oak, orange, banyan and amla in order, the figure so formed is a regular pentagon.

Planting more trees helpful in reducing pollution and make the environment clean and green for the coming generations.

Assertion and Reason Answers-

1. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation:

Assertion (A) :

The points (–3, 5) and (5, –3) are at different positions in the coordinate plane.

For the point (–3, 5)

Abscissa = –3 & ordinate = 5

The point lies is 2^{nd} quadrant

For the point (5, –3)

Abscissa = 5 & ordinate = –3

The point lies is 4^{th} quadrant

Since $5 \neq -3$

So the points (–3, 5) and (5, –3) are at different positions in the coordinate plane.

So Assertion (A) is correct

Reason(R) :

If x is not equal to y then the position of (x, y) in the cartesian plane is different from the position of (y, x)

We know that two point (a, b) & (c, d) are the same point iff a = c , b = d

So if x is not equal to y then the position of (x, y) in the cartesian plane is different from the position of (y, x)

Therefore Reason(R) is correct

Also reason (R) is the correct explanation of assertion

Hence the correct option is:

(a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. (d) Assertion is wrong statement but reason is correct statement.

Explanation: (–5, 0) lies on x-axis because the first part of co-ordinate shows X-axis and second part show y-axis and also the point (0, –4) lies on y axis not x axis.

Hence the assertion is false but the reason is 100% true statement.

Case Study Answer:

1.

(i)	(b)	(3, 4)
(ii)	(a)	(2, –3)
(iii)	(c)	x-axis.
(iv)	(d)	origin.
(v)	(a)	–3

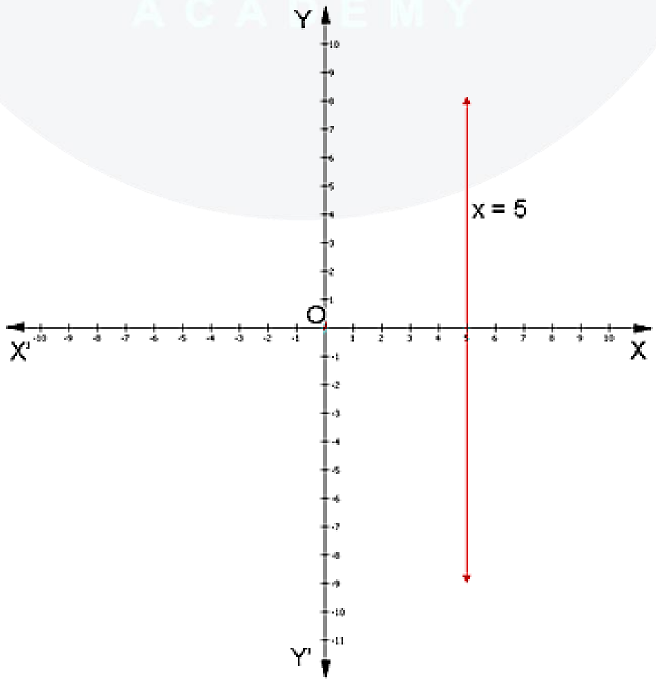
2.

(i)	(d)	6 feet
(ii)	(c)	6 feet
(iii)	(a)	AB and CD
(iv)	(b)	AC and BD
(v)	(a)	1

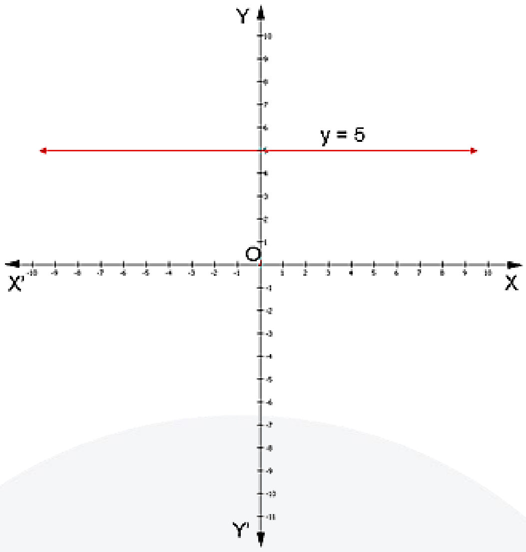
Linear Equations in Two Variables 4

Key Concepts

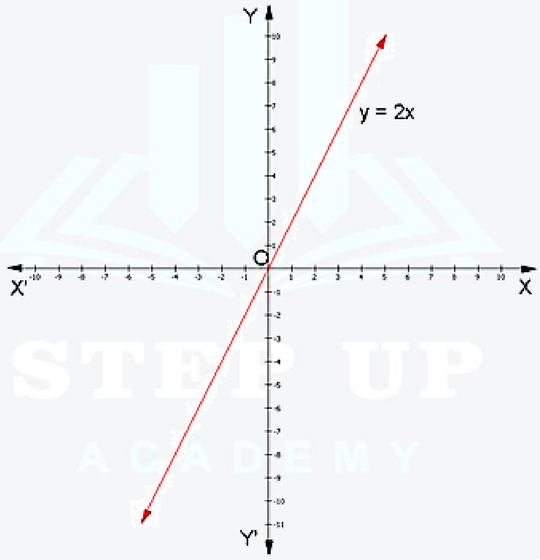
1. An equation of the form ax + by + c = 0, where a, b and c are real numbers, such that a and b are not both zero, is called a **linear equation in two variables**.
2. Linear equations in one variable, of the type ax + b = 0, can also expressed as a linear equation in two variables. Since, ax + b = 0 ⇒ ax + 0.y + b = 0.
3. A **solution** of a linear equation in two variables is a pair of values, one for x and one for y, which satisfy the equation.
4. The solution of a linear equation is not affected when-
   i. The same number is added or subtracted from both the sides of an equation.
   ii. Multiplying or dividing both the sides of the equation by the same non-zero number.
5. A linear equation in two variables has **infinitely many solutions**.
6. Every point on the line satisfies the equation of the line and every solution of the equation is a point on the line.
7. A linear equation in two variables is represented geometrically by a straight line whose points make up the collection of solutions of the equation. This is called the **graph** of the linear equation.
8. x = 0 is the equation of the y-axis and y = 0 is the equation of the x-axis.
9. The graph of **x = k** is a straight line parallel to the y-axis.
   For example, the graph of the equation x = 5 is as follows:



10. The graph of **y = k** is a straight line parallel to the x-axis.
For example, the graph of the equation y = 5 is as follows:



11. An equation of the type **y = mx** represents a line passing through the origin, where m is a real number. For example, the graph of the equation y = 2x is as follows:



Linear equation in one variable

When an equation has only one variable of degree one, then that equation is known as linear equation in one variable.

A linear equation in one variable is an equation which has a maximum of one variable of order 1. It is of the form ax + b = 0, where x is the variable.

- Standard form: ax + b = 0, where a and b ϵ R & a ≠ 0
- Examples of linear equation in one variable are:
  – 3x – 9 = 0
  – 2t = 5

Standard Form of Linear Equations in One Variable

The standard form of linear equations in one variable is represented as:

ax + b = 0

Where,

- 'a' and 'b' are real numbers.
- Both 'a' and 'b' are not equal to zero.

Thus, the formula of linear equation in one variable is ax + b = 0.

Solving Linear Equations in One Variable

For solving an equation having only one variable, the following steps are followed

- Step 1: Using LCM, clear the fractions if any.
- Step 2: Simplify both sides of the equation.
- Step 3: Isolate the variable.
- Step 4: Verify your answer.

Example of Solution of Linear Equation in One Variable

Let us understand the concept with the help of an example.

For solving equations with variables on both sides, the following steps are followed:

Consider the equation: $5x - 9 = -3x + 19$

Step 1: Transpose all the variables on one side of the equation. By transpose, we mean to shift the variables from one side of the equation to the other side of the equation. In the method of transposition, the operation on the operand gets reversed.

In the equation $5x - 9 = -3x + 19$, we transpose -3x from the right-hand side to the left-hand side of the equality, the operation gets reversed upon transposition and the equation becomes:

$5x - 9 + 3x = 19$

$\Rightarrow 8x - 9 = 19$

Step 2: Similarly transpose all the constant terms on the other side of the equation as below:

$8x - 9 = 19$

$\Rightarrow 8x = 19 + 9$

$\Rightarrow 8x = 28$

Step 3: Divide the equation with 8 on both sides of the equality.

$8x/8 = 28/8$

$\Rightarrow x = 28/8$

If we substitute $x = 28/8$ in the equation $5x - 9 = -3x + 19$, we will get 9 = 9, thereby satisfying the equality and giving us the required solution.

The Application of Linear equation:

There are various applications of linear equations in Mathematics as well as in real life. An algebraic equation is an equality that includes variables and equal sign (=). A linear equation is an equation of degree one.

The knowledge of mathematics is frequently applied through word problems, and the applications of linear equations are observed on a wide scale to solve such word problems. Here, we are going to discuss the linear equation applications and how to use them in the real world with the help of an example.

A linear equation is an algebraic expression with a variable and equality sign (=), and whose highest degree is equal to 1. For example, $2x - 1 = 5$ is a linear equation.

- Linear equation with one variable and degree one is called a linear equation in one variable. (Eg, $3x+5=0$)
- Linear equation with degree one and two variables is called a linear equation in two variables. (Eg, $3x+5y=0$)

The graphical representation of linear equation is $ax + by + c = 0$, where,

- a and b are coefficients
- x and y are variables
- c is a constant term

In real life, the applications of linear equations are vast. To tackle real-life problems using algebra, we convert the given situation into mathematical statements in such a way that it clearly illustrates the relationship between the unknowns (variables) and the information provided. The following steps are involved while restating a situation into a mathematical statement:

- Translate the problem statement into a mathematical statement and set it up in the form of algebraic expression in a manner it illustrates the problem aptly.
- Identify the unknowns in the problem and assign variables (quantity whose value can change depending upon the mathematical context) to these unknown quantities.
- Read the problem thoroughly multiple times and cite the data, phrases and keywords. Organize the information obtained sequentially.
- Frame an equation with the help of the algebraic expression and the data provided in the problem statement and solve it using systematic techniques of equation solving.
- Retrace your solution to the problem statement and analyze if it suits the criterion of the problem.

There you go!! Using these steps and applications of linear equations word problems can be solved easily.

Applications of Linear equations in Real life

- Finding unknown age
- Finding unknown angles in geometry
- For calculation of speed, distance or time
- Problems based on force and pressure

Let us look into an example to analyze the applications of linear equations in depth.

Applications of Linear Equations Solved Example

Example:

Rishi is twice as old as Vani. 10 years ago his age was thrice of Vani. Find their present ages.

Solution:

In this word problem, the ages of Rishi and Vani are unknown quantities. Therefore, as discussed above, let us first choose variables for the unknowns.

Let us assume that Vani's present age is 'x' years. Since Rishi's present age is 2 times that of Vani, therefore his present age can be assumed to be '2x'.

10 years ago, Vani's age would have been 'x – 10 ', and Rishi's age would have been '2x – 10'. According to the problem statement, 10 years ago, Rishi's age was thrice of Vani, i.e. $2x - 10 = 3(x - 10)$.

We have our linear equation in the variable 'x' which clearly defines the problem statement. Now we can solve this linear equation easily and get the result.

$2x - 10 = 3(x - 10)$

$\Rightarrow 2x - 10 = 3x - 30$

$\Rightarrow x = 20$

This implies that the current age of Vani is 20 years, and Rishi's age is '2x,' i.e. 40 years. Let us retrace our solution. If the present age of Vani is 20 years then 10 years ago her age would have been 10 years, and Rishi's age would have been 30 years which satisfies our problem statement. Thus, applications of linear equations enable us to tackle such real-world problems.

Linear Equations Formula

A linear equation looks like any other equation. It is made up of two expressions set equal to each other. It is equal to the product that is directly proportional to the other plus the constant.

The Linear equation formula is given by

$y = mx + b$

Where,

m determines the slope of that line,

b determines the point at which the line crosses

Solved Examples

Question 1: Solve for x: 5x + 6 = 11

Solution:

Given function is 5x + 6 = 11

5x = 11 – 6

$x = \frac{5}{5} = 1$

Therefore, x = 1.

Graphing of Linear Equations

Linear equations, also known as first-order degree equations, where the highest power of the variable is one. When an equation has one variable, it is known as linear equations in one variable. If the linear equations contain two variables, then it is known as linear equations in two variables, and so on. In this article, we are going to discuss the linear equations in two variables, and also going to learn about the graphing of linear equations in two variables with examples.

Linear Equations in Two Variables

Equations of degree one and having two variables are known as linear equations in two variables. It is of the form, ax + by + c = 0, where a, b and c are real numbers, and both a and b not equal to zero.

Equations of the form ax + by = 0; where a and b are real numbers, and a, b ≠ 0, is also linear equations in two variable.

Solution of a Linear Equation in Two Variables

The solution of a linear equation in two variables is a pair of numbers, one for x and one for y which satisfies the equation. There are infinitely many solutions for a linear equation in two variables.

For example, x + 2y = 6 is a linear equation and some of its solution are (0, 3), (6, 0), (2, 2) because, they satisfy x + 2y = 6.

Graphing of Linear Equation in Two Variables

Since the solution of linear equation in two variable is a pair of numbers (x, y), we can represent the solutions in a coordinate plane.

Consider the equation,

2x + y = 6 ……..(1)

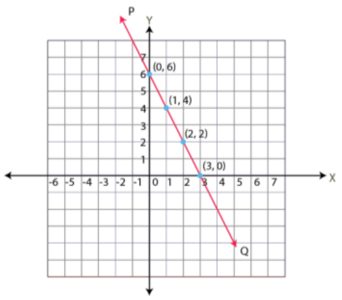
Some solutions of the above equation are, (0, 6), (3, 0), (1, 4), (2, 2) because, they satisfy (1).

We can represent the solution of (1) using a table as shown below.

x	0	3	1	2	…
Y	6	0	4	2	…

We can plot the above points (0, 6), (3, 0), (1, 4), (2, 2) in a coordinate plane (Refer figure).

We can take any two points and join those to make a line. Let the line be PQ. It is observed that all the four points are lying on the same line PQ.



Consider any other point on the line PQ, for example, take point (4, -2) which lies on PQ.

Let's check whether this point satisfies the equation or not.

Substituting (4, –2) in (1) gives,

LHS = (2 × 4) – 2 = 6 = RHS

Therefore (4, –2) is a solution of (1).

Similarly, if we take any point on the line PQ, it will satisfy (1).

It can be observed that,

- All the points say, (p, q) on the line PQ gives a solution of 2x + y = 6.
- All the solution of 2x + y = 6, lie on the line PQ.
- Points which are not the solution of 2x + y = 6 will not lie on the line PQ.

Graphing of Linear Equations key points

It can be concluded that, for a linear equation in two variables,

- Every point on the line will be a solution to the equation.
- Every solution of the equation will be a point on the line.

Therefore, every linear equation in two variables can be represented geometrically as a straight line in a coordinate plane. Points on the line are the solution of the equation. This why equations with degree one are called as linear equations. This representation of a linear equation is known as graphing of linear equations in two variables.

Linear equation in 2 variables

When an equation has two variables both of degree one, then that equation is known as linear equation in two variables.

Standard form: ax + by + c = 0, where a, b, c ϵ R & a, b ≠ 0

Examples of linear equations in two variables are:

– 7x + y = 8

– 6p – 4q + 12 = 0

Examples of Linear Equations

The solution of linear equation in 2 variables

A linear equation in two variables has a pair of numbers that can satisfy the equation. This pair of numbers is called as the solution of the linear equation in two variables.

The solution can be found by assuming the value of one of the variable and then proceed to find the other solution.

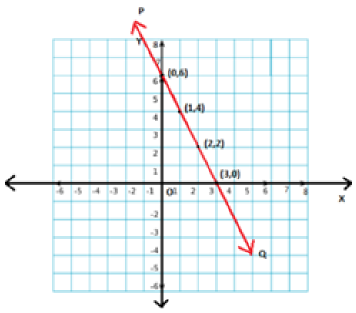
There are infinitely many solutions for a single linear equation in two variables.

Graph of a Linear Equation

Graphical representation of a linear equation in 2 variables

Any linear equation in the standard form ax + by + c = 0 has a pair of solutions in the form (x, y), that can be represented in the coordinate plane.

When an equation is represented graphically, it is a straight line that may or may not cut the coordinate axes.



Solutions of Linear equation in 2 variables on a graph

- A linear equation $ax + by + c = 0$ is represented graphically as a straight line.
- Every point on the line is a solution for the linear equation.
- Every solution of the linear equation is a point on the line.

Lines passing through the origin

- Certain linear equations exist such that their solution is (0, 0). Such equations when represented graphically pass through the origin.
- The coordinate axes namely x-axis and y-axis can be represented as $y=0$ and $x=0$, respectively.

Lines parallel to coordinate axes

- Linear equations of the form $y = a$, when represented graphically are lines parallel to the x-axis and a is the y-coordinate of the points in that line.
- Linear equations of the form $x = a$, when represented graphically are lines parallel to the y-axis and a is the x-coordinate of the points I n that line.

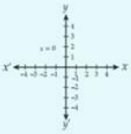
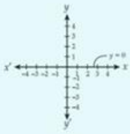
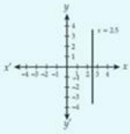
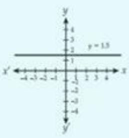
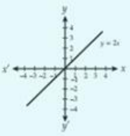
Class : 9th mathematics
Chapter- 4: Linear Equations in Two Variables

Linear Equations in Two Variables

Linear equation
- Equation of the form ax + by + c = 0
- Where → a , b c - constants, $(a,b) \neq (0,0)$
- Example → x, y - variables

Steps to find solution
- **Step 1:** Write the equation in two variables, if not present
- **Step 2:** Reduce it to one variable by putting an arbitrary value of any variable, to find a pair of solution.
- **Step 3:** Repeat step 2 for another arbitrary value of variable to find another pair of solution.
- It can have one, no or infinitely many solutions.

Graphical Representation

Equation	Interpretation	Graphical representation
$x = 0$	Equation of y-axis	
$y = 0$	Equation of x-axis	
$x = K$	Straight line parallel to y-axis	
$y = K$	Straight line parallel to x-axis	
$y = mx$	Line passing through origin	

Example - Graph of Linear Equation

Linear equation: $2x + 3y + 12$

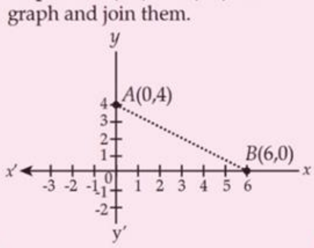
Step 1: From equation, we get

$$y = \frac{12 - 2x}{3}$$

Step 2: Put arbitrary value of x, y.

x	0	0
y	4	6

Step 3: Plot (0,4) and (6,0) on the graph and join them.



Important Questions

Multiple Choice Questions

1. The linear equation 3x – 11y = 10 has:
   (a) Unique solution
   (b) Two solutions
   (c) Infinitely many solutions
   (d) No solutions
2. 3x + 10 = 0 will has:
   (a) Unique solution
   (b) Two solutions
   (c) Infinitely many solutions
   (d) No solutions
3. The solution of equation x – 2y = 4 is:
   (a) (0,2)
   (b) (2,0)
   (c) (4,0)
   (d) (1,1)
4. The value of k, if x = 1, y = 2 is a solution of the equation 2x + 3y = k.
   (a) 5
   (b) 6
   (c) 7
   (d) 8
5. Point (3, 4) lies on the graph of the equation 3y = kx + 7. The value of k is:
   (a) 4/3
   (b) 5/3
   (c) 3
   (d) 7/3
6. The graph of linear equation x + 2y = 2, cuts the y-axis at:
   (a) (2, 0)
   (b) (0, 2)
   (c) (0, 1)
   (d) (1, 1)
7. Any point on the line x = y is of the form:
   (a) (k, –k)
   (b) (0, k)
   (c) (k, 0)
   (d) (k, k)
8. The graph of x = 3 is a line:
   (a) Parallel to x-axis at a distance of 3 units from the origin
   (b) Parallel to y-axis at a distance of 3 units from the origin
   (c) Makes an intercept 3 on x-axis
   (d) Makes an intercept 3 on y-axis
9. In equation, y = mx + c, m is:
   (a) Intercept
   (b) Slope of the line
   (c) Solution of the equation
   (d) None of the above
10. If x and y are both positive solutions of equation ax + by + c = 0, always lie in:
   (a) First quadrant
   (b) Second quadrant
   (c) Third quadrant
   (d) Fourth quadrant

Very Short Questions:

1. Linear equation x – 2 = 0 is parallel to which axis?
2. Express x in term of y: $\frac{x}{7}+2y=6$.
3. If we multiply or divide both sides of a linear equation with a non-zero number, then what will happen to the solution of the linear equation?
4. Find the value of k for which x = 0, y = 8 is a solution of 3x – 6y = k.
5. Write the equation of a line which is parallel to x-axis and is at a distance of 2 units from the origin.
6. Find 'a', if linear equation 3x – ay = 6 has one solution as (4, 3).
7. Cost of a pen is two and half times the cost of a pencil. Express this situation as a linear equation in two variables.
8. In an one day international cricket match, Raina and Dhoni together scored 198 runs. Express the statement as a linear equation in two variables.
9. The cost of a table is 100 more than half the cost of a chair. Write this statement as a linear equation in two variables.

Short Questions:

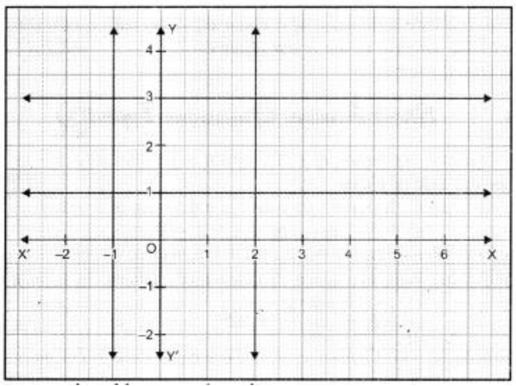
1. Write linear equation representing a line which is parallel to y-axis and is at a distance of 2 units on the left side of y-axis.
2. In some countries temperature is measured in Fahrenheit, whereas in countries like India it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius :

$$F=\left(\frac{9}{5}\right)C+32°$$

   If the temperature is −40°C, then what is the temperature in Fahrenheit?
3. If the temperature is − 0°C, then what is the temperature in Fahrenheit?
4. If ax + 3y = 25, write y in terms of x and also, find the two solutions of this equation.
5. Find the value of k, if (1, −1) is a solution of the equation 3x − ky = 8. Also, find the coordinates of another point lying on its graph.
6. Let y varies directly as x. If y = 12 when x = 4, then write a linear equation. What is the value of y, when x = 5?

Long Questions:

1. Write the equations of the lines drawn in following graph:



   Also, find the area enclosed between these lines.
2. If (2, 3) and (4, 0) lie on the graph of equation ax + by = 1. Find the value of a and b. Plot the graph of equation obtained.
3. Draw the graphs of the following equations on the same graph sheet:

   x = 4, x = 2, y = 1 and y − 3 = 0.
4. Cost of 1 pen is ₹ x and that of 1 pencil is ₹ y. Cost of 2 pens and 3 pencils together is ₹ 18. Write a linear equation which satisfies this data. Draw the graph for the same.
5. Sum of two numbers is 8. Write this in the form of a linear equation in two variables. Also, draw the line given by this equation. Find graphically the numbers, if difference between them is 2.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: There are infinite number of lines which passes through (2, 14).

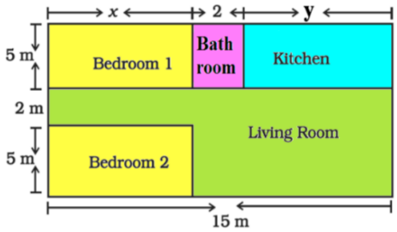
   Reason: A linear equation in two variables has infinitely many solutions.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: All the points (1, 0), (−1, 0), (2, 0) and (5, 0) lie on the x -axis.

   Reason: Equation of the x-axis is y = 0.

Case Study Questions:

1. In the below given layout, the design and measurements has been made such that area of two bedrooms and Kitchen together is 95 sq. m.



i. The area of two bedrooms and kitchen are respectively equal to:
   a. 5x, 5y
   b. 10x, 5y
   c. 5x, 10y
   d. x, y

ii. Find the length of the outer boundary of the layout.
   a. 27m
   b. 15m
   c. 50m
   d. 54m

iii. The pair of linear equation in two variables formed from the statements are
   a. x + y = 13, x + y = 9
   b. 2x + y = 13, x + y = 9
   c. x + y = 13, 2x + y = 9
   d. None of the above

iv. Which is the solution satisfying both the equations formed in (iii)?
   a. x = 7, y = 6
   b. x = 8, y = 5
   c. x = 6, y = 7
   d. x = 5, y = 8

v. Find the area of each bedroom.
   a. 30 sq. m
   b. 35 sq. m
   c. 65 sq. m
   d. 42 sq. m

Answer Key

Multiple Choice Questions

1. (c) Infinitely many solutions
2. (a) Unique solution
3. (c) (4, 0)
4. (d) 8
5. (b) 5/3
6. (c) (0, 1)
7. (d) (k, k)
8. (b) Parallel to y-axis at a distance of 3 units from the origin
9. (b) Slope of the line
10. (a) First quadrant

Very Short Answer:

1. Here, linear equation is x – 2 ⇒ 0 x = 2

   Thus, it is parallel to the y-axis.

2. Given equation is

   $$\frac{x}{7}+2y=6$$

   $$\Rightarrow \frac{x}{7}=6-2y$$

   Thus, x = 7(6 – 2y).

3. Solution remains the same.

4. Since x = 0 and y = 8 is a solution of given equation

   3x – 6y = k

   3(0) – 6(8) = k

   ⇒ k = – 48

5. Here, required line is parallel to x-axis and at a distance of 2 units from the origin.

   ∴ Its equation is

   y + 2 = 0

   or y – 2 = 0

6. Since (4, 3) is a solution of given equation.

   ∴ 3(4) – a(3) = 6

   ⇒ 12 – 3a = 6

   $$\Rightarrow a=\frac{-6}{-3}$$

   Hence, a = 2

7. Let cost of a pen be ₹ x and cost of a pencil be ₹ y.

   According to statement of the question, we have

   $$x=2\frac{1}{2}y \Rightarrow 2x = 5y \text{ or } 2x - 5y = 0$$

8. Let runs scored by Raina be x and runs scored by Dhoni be y.

According to statement of the question, we have

$x + y = 198$

$x + y - 198 = 0$

9. Let the cost price of a table be ₹ x and that of a chair be ₹ y.

$\therefore x = \frac{1}{2}y + 100$

$\Rightarrow 2x = y + 200$ or $2x - y - 200 = 0$.

Short Answer:

1. **Solution:**

Here, required equation is parallel to y-axis at a distance of 2 units on the left side of y-axis.

$x = -2$ or $x + 2 = 0$

2. **Solution:**

Given linear equation is $F = \left(\frac{9}{5}\right)C + 32°$

Put C = –40°, we have

$F = \frac{9}{5}(-40°) + 32°$

$F = -72° + 32°$

$F = -40°$

3. **Solution:**

Since there are infinite lines passing through the point (2, 3).

Let, first equation is $x + y = 5$ and second equation is $2x + 3y = 13$.

Clearly, the lines represented by both equations intersect at the point (2, 3).

4. **Solution:**

Given equation is $\pi x + 3y = 25$

$\therefore \quad y = \frac{25 - \pi x}{3}$

When x = 0, then $y = \frac{25}{3}$

When x = 1, then $y = \frac{25 - \pi}{3}$

Hence, the two solutions are $x = 0$, $y = \frac{25}{3}$ and $y = \frac{25 - \pi}{3}$.

5. **Solution:**

Since (1,–1) is a solution of the equation $3x - ky = 8$

$\therefore 3(1) - k(-1) = 8$

$\Rightarrow k = 8 - 3 = 5$

Thus, the given equation is

$3x - 5y = 8$

Put x = 6, then $y = \frac{3 \times 6 - 8}{5} = \frac{18 - 8}{5} = \frac{10}{5} = 2$

Hence, the coordinates of another point lying on the graph of $3x - 5y = 8$ is (6, 2).

6. **Solution:**

Given y varies directly as x implies $y = kx$

But y = 12 for x = 4

$\Rightarrow 4k = 12 = k = 3$

Put k = 3 in y = kx, we have

$y = 3x$

Now, when x = 5, y = 3 x 5 = y = 15 ...(i)

7. **Solution:**

Let numerator and denominator of the given fraction be respectively x and y. According to the statement, we obtain

$\frac{x - 2}{y + 3} = \frac{1}{4}$

$\Rightarrow 4x - 8 = y + 3$

$\Rightarrow 4x - y - 11 = 0$

Which is the required linear equation. When y = 1, then x = 3. When y = 5, then x = 4. Hence, the two solutions are (3, 1) and (4, 5).

Long Answer:

1. **Solution:**

Equations of the lines drawn in the graph are as :

$x = -1$ or $x + 1 = 0$,

$x = 2$ or $x - 2 = 0$,

$y = 1$ or $y - 1 = 0$ and

$y = 3$ or $y - 3 = 0$

Figure formed by these lines is a rectangle of dimensions 3 units by 2 units.

Hence, the area enclosed between given lines = 6 sq. units.

2. **Solution:**

(2, 3) and (4, 0) lie on the graph of equation

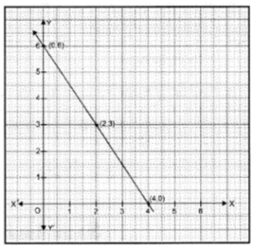
$ax + by = 1$ ...(i)

$\therefore$ We have $2a + 3b = 1$ ... (ii)

and $4a + 0 = 1$

$\Rightarrow \quad a = \frac{1}{4}$

Putting the value of a in eq. (ii), we have



$2\times\frac{1}{4}+3b=1$

$\frac{1}{2}+3b=1$

$\Rightarrow \quad 3b=\frac{1}{2}$

$\Rightarrow \quad b=\frac{1}{6}$

Putting the value of a and b in eq. (i), we have

$\frac{1}{4}+\frac{1}{6}y=1$

$\Rightarrow \quad \frac{3x+2y}{12}=1 \Rightarrow \quad 3x+2y=12$ ...(iii)

Which is required linear equation.

Put x= 0 in eq. (iii), we have

$\Rightarrow 3(0) + 2y = 12$

$\Rightarrow 2y = 12$

$\Rightarrow y = 6$

Put x = 2 in eq. (iii), we have

$\Rightarrow 3(2) + 2y = 12$

$\Rightarrow 2y = 6$

$\Rightarrow y = 3$

Put x = 4 in eq. (iii), we have

$\Rightarrow 3(4) + 2y = 12$

$\Rightarrow 2y = 0$

$\Rightarrow y = 0$

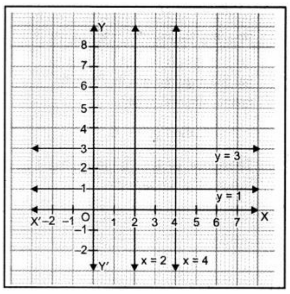
We have the following table:

x	0	2	4
y	6	3	0

By plotting the points (0, 6), (2, 3) and (4, 0). Joining them, we obtained the graph of 3x+2y=12.

3. **Solution:**

By plotting the points (0, 6), (2, 3) and (4, 0). Joining them, we obtained the graph of 3x+2y=12.



4. **Solution:**

Here, cost of 1 pen is ₹x and that of 1 pencil is ₹ y

According to the statement of the question, we have

$2x + 3y = 18$

$x=\frac{18-3y}{2}$

When y = 0, x = 9

When y = 4, x= 3.

When y = 6, x = 0

Table of solutions is

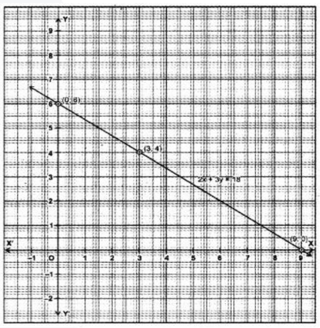
x	0	3	9
y	6	4	0

Plot the points (0, 6), (3, 4) and (9, 0). Join them in pairs to get the required line.

5. **Solution:**

Let the two numbers be x and y.

It is given that sum of two numbers is 8.



$\therefore x + y = 8$

$y = 8 - x$

When x = 0,

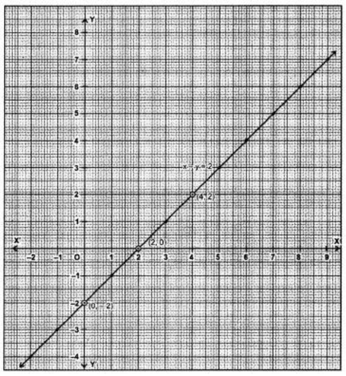
When x = 4, y = 4

When x = 8, y = 0

Table of solutions is

x	0	4	8
y	8	4	0

Plot the points (0, 8), (4, 4), (8, 0) and join them in pairs, we get the required graph.



When difference between two number is 2, then

$x - y = 2, x > y$

$\Rightarrow x = y + 2$

When x = 0, y = –2

When x = 2, y = 0

When x = 4, y = 2

Table of solutions is:

x	0	2	4
y	–2	0	2

Plot these points (0, –2), (2, 0), (4, 2) and join them to get the required line.

Graphically, the numbers are: (–2, 4), (–1, –3), (0, –2), (1, –1), (2, 0), (3, 1), (4, 2), (5, 3), (6, 4), (7, 5) etc.

Assertion and Reason Answers-

1. (b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation:

Assertion : There are infinite number of lines which passes through (2 , 14) .

For a given point there can be infinite number of line passing through.

Hence Assertion is true to Define one line, there should be atleast 2 distinct points.

Reason : A linear equation in two variables has infinitely many solutions.

ax + by = c

Has infinitely many solutions as infinite point lies on a line.

Hence Reason is True.

But Reason is not the correct explanation of assertion as reason is about infinite points on a given line while assertion is about infinite lines passing through a point.

Hence,

Both assertion and reason are true but reason is not the correct explanation of assertion.

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: Points on x-axis have '0' as their ordinate.

So, (1, 0), (–1, 0), (2, 0) and (5, 0) all lie on x-axis.

Equation of x-axis is y = 0

∴ Both assertion and reason are correct.

Case Study Answer:

i. (b) 10x, 5y

Explanation:

Area of one bedroom = 5x sq.m

Area of two bedrooms = 10x sq.m

Area of kitchen = 5y sq. m

ii. (d) 54m

Explanation:

Length of outer boundary = 12 + 15 + 12 + 15 = 54 m

iii. (d) None of the above

Explanation:

Area of two bedrooms = 10x sq.m

Area of kitchen = 5y sq. m

So, 10x + 5y = 95, 2x + y = 19

Also, x + 2 + y = 15, x + y = 13

iv. x = 6, y = 7

Explanation:

x + y = 6 + 7 = 13

2x + y = 2(6) + 7 = 19

x = 6, y = 7

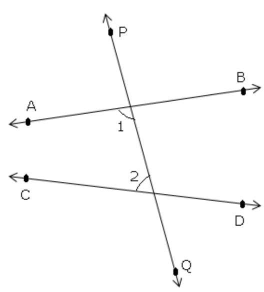
v. (a) 30 sq. m

Introduction to Euclid's Geometry 5

Key Concepts

1. A **point** is that which has no part.
2. A **line** is a breadthless length. The ends of a line are points.
3. A **straight line** is a line which lies evenly with the points on itself.
4. A **surface** is that which has length and breadth only.
5. The **edges** of a surface are lines.
6. A **plane surface** is a surface which lies evenly with the straight lines on itself.
7. Though Euclid defined **a point, a line and a plane,** but the definitions are not accepted by mathematicians. Therefore, these terms are taken as **undefined**.
8. An **axiom** is a statement accepted as true without proof, throughout mathematics.
9. A **postulate** is a statement accepted as true without proof, specifically in geometry.
10. **Euclid's Axioms:**
    i. Things which are equal to the same things are equal to one another.
    ii. If equals are added to equals, the wholes are equal.
    iii. If equals are subtracted from equals, the remainders are equal.
    iv. Things which coincide with one another are equal to one another.
    v. The whole is greater than a part.
    vi. Things which are double of same things are equal to one another.
    vii. Things which are halves of same things are equal to one another.
11. A > B means that there is some quantity C such that A = B + C.
12. **Theorems** are statements which are proved using definitions, axioms, previously proved statement and deductive reasoning.
13. Euclid's 5 Postulates:
    i. A straight line may be drawn from any one point to any other point.
    ii. A terminated line can be produced indefinitely.

    iii. A circle can be drawn with any centre and any radius.
    iv. All right angles are equal to one another.
    v. If a straight line falling on two straight lines makes the interior angles on the same side of it taken together, is less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.



This is known as the parallel postulate.

In the figure, $\angle 1 + \angle 2 < 180°$. The lines AB and CD will eventually intersect on the left side of PQ.

14. Given two distinct points, there is a unique line that passes through them.
15. Two distinct lines cannot have more than one point in common.
16. **Equivalent version of Euclid's fifth postulate:**
    i. For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.
       This result is also known as 'Playfair's Axiom'.
    ii. Two distinct intersecting lines cannot be parallel to the same line.
17. All attempts to prove Euclid's fifth postulate using first four postulates failed and led to several other geometries called non-Euclidean geometries.
18. The distance of a point from a line is the length of the perpendicular from the point to the line.

Euclid's Geometry

The word "geometry" comes from the Greek words "geo", which means the "earth", and "metron", which means "to measure". Euclidean geometry is a mathematical system attributed to Euclid a teacher of mathematics in Alexandria in Egypt. Euclid gave us an exceptional idea regarding the basic concepts of geometry, in his book called "Elements".

Euclid listed 23 definitions in his book "Elements". Some important points are mentioned below:

- A line is an endless length.
- A point has no dimension (length, breadth and width).
- A line which lies evenly with the points on itself is a straight line.
- Points are the ends of a line.
- A surface is that which has breadth and length only.
- A plane surface is a surface which lies evenly with the straight lines on itself.
- Lines are the edges of a surface.

Euclid realized that a precise development of geometry must start with the foundations. Euclid's axioms and postulates are still studied for a better understanding of geometry.

Euclidean Geometry

Euclidean geometry is the study of geometrical shapes (plane and solid) and figures based on different axioms and theorems. It is basically introduced for flat surfaces or plane surfaces. Geometry is derived from the Greek words 'geo' which means earth and 'metrein' which means 'to measure'.

Euclidean geometry is better explained especially for the shapes of geometrical figures and planes. This part of geometry was employed by the Greek mathematician Euclid, who has also described it in his book, Elements. Therefore, this geometry is also called Euclid geometry.

The axioms or postulates are the assumptions that are obvious universal truths, they are not proved. Euclid has introduced the geometry fundamentals like geometric shapes and figures in his book elements and has stated 5 main axioms or postulates. Here, we are going to discuss the definition of euclidean geometry, its elements, axioms and five important postulates.

History of Euclid Geometry

The excavations at Harappa and Mohenjo-Daro depict the extremely well-planned towns of Indus Valley Civilization (about 3300-1300 BC). The flawless construction of Pyramids by the Egyptians is yet another example of extensive use of geometrical techniques used by the people back then. In India, the Sulba Sutras, textbooks on Geometry depict that the Indian Vedic Period had a tradition of Geometry.

The development of geometry was taking place gradually, when Euclid, a teacher of mathematics, at Alexandria in Egypt, collected most of these evolutions in geometry and compiled it into his famous treatise, which he named 'Elements'.

Euclidean Geometry

Euclidean Geometry is considered an axiomatic system, where all the theorems are derived from a small number of simple axioms. Since the term "Geometry" deals with things like points, lines, angles, squares, triangles, and other shapes, Euclidean Geometry is also known as "plane geometry". It deals with the properties and relationships between all things.

Plane Geometry	Solid Geometry
1. Congruence of triangles 2. Similarity of triangles 3. Areas 4. Pythagorean theorem 5. Circles 6. Regular polygons 7. Conic sections	1. Volume 2. Regular solids

Examples of Euclidean Geometry

The two common examples of Euclidean geometry are angles and circles. Angles are said as the inclination of two straight lines. A circle is a plane figure, that has all the points at a constant distance (called the radius) from the center.

Euclidean and Non-Euclidean Geometry

There is a difference between Euclidean and non-Euclidean geometry in the nature of parallel lines. In Euclidean geometry, for the given point and line, there is exactly a single line that passes through the given points in the same plane and it never intersects.

Non-Euclidean is different from Euclidean geometry. The spherical geometry is an example of non-Euclidean geometry because lines are not straight here.

Properties of Euclidean Geometry

- It is the study of plane geometry and solid geometry
- It defined point, line and a plane
- A solid has shape, size, position, and can be moved from one place to another.
- The interior angles of a triangle add up to 180 degrees
- Two parallel lines never cross each other
- The shortest distance between two points is always a straight line

Elements in Euclidean Geometry

In Euclidean geometry, Euclid's Elements is a mathematical and geometrical work consisting of 13 books written by ancient Greek mathematician Euclid in Alexandria, Ptolemaic Egypt. Further, the 'Elements' was divided into thirteen books that popularized geometry all over the world. As a whole, these Elements is a collection of definitions, postulates (axioms), propositions (theorems and constructions), and mathematical proofs of the propositions.

Book 1 to 4th and 6th discuss plane geometry. He gave five postulates for plane geometry known as Euclid's Postulates and the geometry is known as Euclidean geometry. It was through his works, we have a collective source for learning geometry; it lays the foundation for geometry as we know it now.

Euclidean Axioms

Here are the seven axioms are given by Euclid for geometry.

- Things which are equal to the same thing are equal to one another.
- If equals are added to equals, the wholes are equal.
- If equals are subtracted from equals, the remainders are equal.

- Things which coincide with one another are equal to one another.
- The whole is greater than the part.
- Things which are double of the same things are equal to one another.
- Things which are halves of the same things are equal to one another.

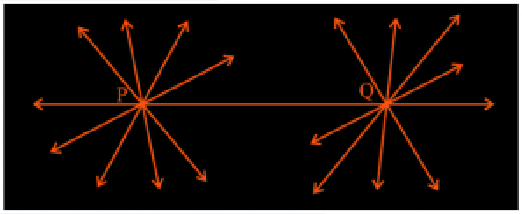
Euclid's Five Postulates

Before discussing Postulates in Euclidean geometry, let us discuss a few terms as listed by Euclid in his book 1 of the 'Elements'. The postulated statements of these are:

- Assume the three steps from solids to points as solids-surface-lines-points. In each step, one dimension is lost.
- A solid has 3 dimensions, the surface has 2, the line has 1 and the point is dimensionless.
- A point is anything that has no part, a breadthless length is a line and the ends of a line point.
- A surface is something that has length and breadth only.

It can be seen that the definition of a few terms needs extra specification. Now let us discuss these Postulates in detail.

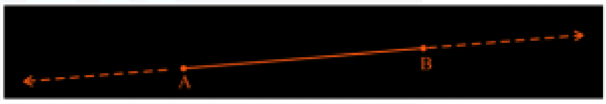
Euclid's Postulate 1



"A straight line can be drawn from any one point to another point."

This postulate states that at least one straight line passes through two distinct points but he did not mention that there cannot be more than one such line. Although throughout his work he has assumed there exists only a unique line passing through two points.

Euclid's Postulate 2



"A terminated line can be further produced indefinitely."

In simple words what we call a line segment was defined as a terminated line by Euclid. Therefore this postulate means that we can extend a terminated line or a line segment in either direction to form a line. In the figure given below, the line segment AB can be extended as shown to form a line.

Euclid's Postulate 3

"A circle can be drawn with any centre and any radius."

Any circle can be drawn from the end or start point of a circle and the diameter of the circle will be the length of the line segment.

Euclid's Postulate 4

"All right angles are equal to one another."

All the right angles (i.e. angles whose measure is 90°) are always congruent to each other i.e. they are equal irrespective of the length of the sides or their orientations.

Euclid's Postulate 5

"If a straight line falling on two other straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is less than two right angles."

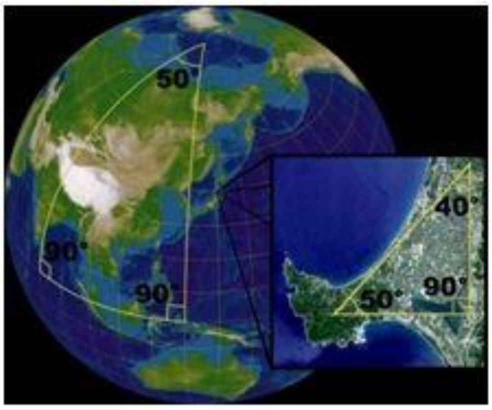
Further, these Postulates and axioms were used by him to prove other geometrical concepts using deductive reasoning. No doubt the foundation of present-day geometry was laid by him and his book the 'Elements'.

Geometry In Daily Life

Geometry is an ancient science and an important branch of mathematics. The ancient mathematician Euclid is credited as the Father of Geometry; who used the word formally in his book "Elements". It is derived from the old

Greek word Geometron meaning to measure the earth (geo: earth and metron: measurement). For an elementary or middle school student, it is all about different basic shapes, including their naming, properties and formulas related to their areas and volumes. But modern geometry has diverged much more from these basic concepts. But none of these has changed their existence and applications in daily life, and it still reflects in our everyday experience.

Applications of Geometry in Everyday Life



Geometry is the most influential branch of mathematics. A keen observation will give you many examples. It was moulded up in ancient era; hence its impact on life is also wide. It's a potential problem solver, especially in practical life. Its applications began long back during Egyptian civilization. They used geometry in different fields such as in art, measurement and architecture. Glorious temples, palaces, dams and bridges are the results of these. In addition to construction and measurements, it has influenced many more fields of engineering, biochemical modelling, designing, computer graphics, and typography.

Daily, we do a lot of tasks with the help of geometry. Some of the common applications include measurement of a line and surface area of land, wrapping gifts, filling of a box or tiffin without overflow, shapes used for different signboards. A person with good practical knowledge of geometry can help himself to measure the dimensions of a land without a chance of conflict. Other advanced applications include robotics, fashion designing, computer graphics and modelling. For example, in fashion designing, a fashion designer has to know about different shapes and their symmetry for developing the best design.

Geometry Symbols

Geometry Symbols: Geometry is a branch of mathematics that deals with the properties of configurations of geometric objects – (straight) lines, circles and points being the most basic.

The area of mathematics that deals with space, lines, shapes and points

- Plane Geometry is about flat shapes like triangles, circles, and lines,
- Solid Geometry is about solid (3-dimensional) shapes like spheres and cubes.

Geometry Symbol Chart

Let's explore the typical Geometry symbols and meanings used in both basic Geometry and more advanced levels through this geometry symbol chart.

Symbol	Symbol Name	Meaning/definition of the Symbols	Example
∠	angle	formed by two rays	∠ABC = 30º
∟	right angle	= 90°	α = 90°
∢	spherical angle		AOB = 30°
′	arcminute	1° = 60′	α = 60°59′
°	degree	1 turn = 360°	α = 60°
″	arcsecond	1′ = 60″	α = 60°59′59″
$\overrightarrow{AB}$<	ray	line that start from point A	
AB	line segment	the line from point A to point B	
⊥	perpendicular	perpendicular lines (90° angle)	AC ⊥ BC
≅	congruent to	equivalence of geometric shapes and size	∆ABC ≅∆XYZ
||	parallel	parallel lines	AB || CD
∆	triangle	triangle shape	∆ABC ≅∆BCD
~	similarity	same shapes, not the same size	∆ABC ~∆XYZ

π	pi constant	π = 3.141592654... is the ratio between the circumference and diameter of a circle	c = π·d = 2·π·r
|x−y|	distance	distance between points x and y	| x−y | = 5
grad	grads	grads angle unit	360° = 400 grad
rad	radians	radians angle unit	360° = 2π rad

The symbols for angles and triangles are most important and frequently used symbols in geometry.

Analytic Geometry

Analytic Geometry is a branch of algebra, a great invention of Descartes and Fermat, which deals with the modelling of some geometrical objects, such as lines, points, curves, and so on. It is a mathematical subject that uses algebraic symbolism and methods to solve the problems. It establishes the correspondence between the algebraic equations and the geometric curves. The alternate term which is used to represent the analytic geometry is "coordinate Geometry".

It covers some important topics such as midpoints and distance, parallel and perpendicular lines on the coordinate plane, dividing line segments, distance between the line and a point, and so on. The study of analytic geometry is important as it gives the knowledge for the next level of mathematics. It is the traditional way of learning the logical thinking and the problem solving skills. In this article, let us discuss the terms used in the analytic geometry, formulas, Cartesian plane, analytic geometry in three dimensions, its applications, and some solved problems.

Planes

To understand how analytic geometry is important and useful, First, We need to learn what a plane is? If a flat surface goes on infinitely in both the directions, it is called a Plane. So, if you find any point on this plane, it is easy to locate it using Analytic Geometry. You just need to know the coordinates of the point in X and Y plane.

Coordinates

Coordinates are the two ordered pair, which defines the location of any given point in a plane. Let's understand it with the help of the box below.

	A	B	C
1			
2		X	
3			

In the above grid, the columns are labelled as A, B, C, and the rows are labelled as 1, 2, 3.

The location of letter x is B2 i.e. Column B and row 2. So, B and 2 are the coordinates of this box, x.

As there are several boxes in every column and rows, but only one box has the point x, and we can find its location by locating the intersection of row and column of that box. There are different types of coordinates in analytical geometry. Some of them are as follows:

- Cartesian Coordinates
- Polar Coordinates
- Cylindrical Coordinates
- Spherical Coordinates

Let us discuss all these types of coordinates are here in brief.

Cartesian Coordinates

The most well-known coordinate system is the Cartesian coordinate to use, where every point has an x-coordinate and y-coordinate expressing its horizontal position, and vertical position respectively. They are usually addressed as an ordered pair and denoted as (x, y). We can also use this system for three-dimensional geometry, where every point is represented by an ordered triple of coordinates (x, y, z) in Euclidean space.

Polar Coordinates

In the case of polar coordinates, each point in a plane is denoted by the distance 'r' from the origin and the angle θ from the polar axis.

Cylindrical Coordinates

In the case of cylindrical coordinates, all the points are represented by their height, radius from z-axis and the angle projected on the xy-plane with respect to the horizontal axis. The height, radius and the angle are denoted by h, r and θ, respectively.

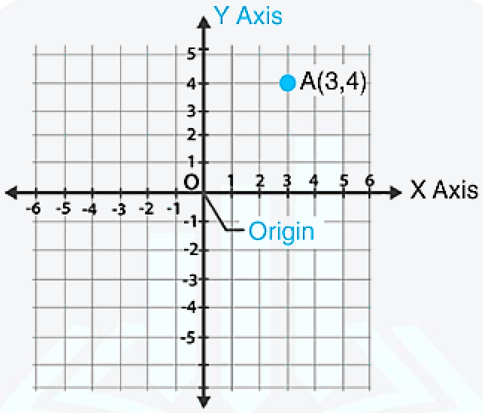
Spherical Coordinates

In spherical coordinates, the point in space is denoted by its distance from the origin (ρ), the angle projected on the xy-plane with respect to the horizontal axis (θ), and another angle with respect to the z-axis (φ).

Cartesian Plane

In coordinate geometry, every point is said to be located on the coordinate plane or cartesian plane only.

Look at the figure below.



The above graph has x-axis and y-axis as it's Scale. The x-axis is running across the plane and Y-axis is running at the right angle to the x-axis. It is similar to the box explained above.

Let's learn more about Co-ordinates:

Origin: It is the point of intersection of the axis(x-axis and y-axis). Both x and y-axis are zero at this point.

Values of the different sides of the axis

x-axis – The values at the right-hand side of this axis are positive and those on the left-hand side are negative.

y-axis – The values above the origin are positive and below the origin are negative.

To locate a point: We need two numbers to locate a plane in the order of writing the location of X-axis first and Y-axis next. Both will tell the single and unique position on the plane. You need to compulsorily follow the order of the points on the plane i.e., the x coordinate is always the first one from the pair. (x, y).

If you look at the figure above, point A has a value 3 on the x-axis and value 2 on the Y-axis. These are the rectangular coordinates of Point A represented as (3, 2).

Using the Cartesian coordinates, we can define the equation of a straight lines, equation of planes, squares and most frequently in the three dimensional geometry. The main function of the analytic geometry is that it defines and represents the various geometrical shapes in the numerical way. It also extracts the numerical information from the shapes.

Analytic Geometry Formulas

Graphs and coordinates are used to find measurements of geometric figures. There are many important formulas in analytic Geometry. Since science and engineering involves the study of rate of change in varying quantities, it helps to show the relation between the quantities involved. The branch of Mathematics called "calculus" requires the clear understanding of the analytic geometry. Here, some of the important ones are being used to find the distance, slope or to find the equation of the line.

Distance Formula

Let the two points be A and B, having coordinates to be (x_1, y_1) and (x_2, y_2) respectively.

Thus, the distance between two points is given as- $d = \sqrt{\left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]}$

Midpoint Theorem Formula

Let A and B are some points in a plane, which is joined to form a line, having coordinates (x_1, y_1) and (x_2, y_2), respectively. Suppose, M(x, y) is the midpoint of the line connecting the point A and B then its formula is given by;

$$M(x, y) = [(x_1+x_2)/2, (y_1+y_2)/2]$$

Angle Formula

Let two lines have slope m1 and m2 and θ is the angle formed between the two lines A and B, which is represented as; $\tan\theta = (m_1-m_2)/(1+m_1m_2)$

Section Formula

Let two lines A and B have coordinates (x_1, y_1) and (x_2, y_2), respectively. A point P the two lines in the ratio of m:n, then the coordinates of P is given by;

- **When the ratio m:n is internal**

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

- **When the ratio m:n is external**

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Class : 9th mathematics
Chapter- 5: Introduction to Euclid's Geometry Variables

Introduction to Euclid's Geometry Variables

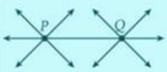
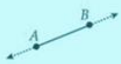
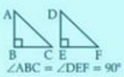
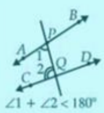
Eucldi 's Geometry

Axiomatic system, in which all theorems are derived from a small number of axioms

Definitions

Point	Has no width, no length and no depth	Point
Line	Collection of points, can be extended in both directions.	Line
Surface	Two - dimensional collection of points (has length & breadth only)	Surface

Euclid 's Postulates

1	A straight line can be drawn from any one point to any other point.	
2	A terminated line can be produced infinitely.	
3	A circle can be drawn with any centre and of any radius.	
4	All right angles are equal to one another.	
5	If a straight line falling on two straight lines makes the interior angles on the same side of it, taken together makes less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of the angles is less than two right angles.	

Euclid's Axioms

1	Things which are equal to the same thing are equal to one another.
2	If equals are added to equals, wholes are equal.
3	If equals are subtracted from equals,the remainders are equal.
4	Things which coincide with one another are equal to one another.
5	The whole is greater than the part.
6	Things which are double of the same things are equal to one another.
7	Things which are halves of the same things are equal to one another.

Important Questions

Multiple Choice Questions

1. Which of the following statements are true?
   (a) Only one line can pass through a single point
   (b) There is an infinite number of lines which pass through two distinct points.
   (c) A terminated line can be produced indefinitely on both the sides.
   (d) If two circles are equal, then their radii are unequal.
2. A solid has ________dimensions.
   (a) One
   (b) Two
   (c) Three
   (d) Zero
3. A point has ______ dimension.:
   (a) One
   (b) Two
   (c) Three
   (d) Zero
4. The shape of base of Pyramid is:
   (a) Triangle
   (b) Square
   (c) Rectangle
   (d) Any polygon
5. The boundaries of solid are called:
   (a) Surfaces
   (b) Curves
   (c) Lines
   (d) Points
6. A surface of a shape has:
   (a) Length, breadth and thickness
   (b) Length and breadth only
   (c) Length and thickness only
   (d) Breadth and thickness only
7. The edges of the surface are:
   (a) Points
   (b) Curves
   (c) Lines
   (d) None of the above
8. Which of these statements do not satisfy Euclid's axiom?
   (a) Things which are equal to the same thing are equal to one another
   (b) If equals are added to equals, the wholes are equal.
   (c) If equals are subtracted from equals, the remainders are equal.
   (d) The whole is lesser than the part.
9. The line drawn from the center of the circle to any point on its circumference is called:
   (a) Radius
   (b) Diameter
   (c) Sector
   (d) Arc
10. There are _______ number of Euclid's Postulates
   (a) Three
   (b) Four
   (c) Five
   (d) Six

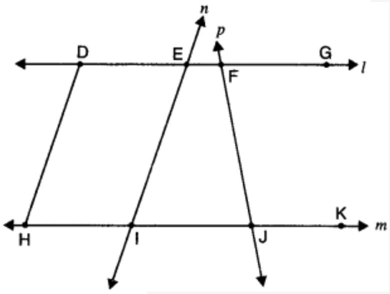
Very Short Questions:

1. Give a definition of parallel lines. Are there other terms that need to be defined first? What are they and how might you define them?
2. Give a definition of perpendicular lines. Are there other terms that need to be defined first? What are they and how might you define them?
3. Give a definition of line segment. Are there other terms that need to be defined first? What are they and how might you define them?
4. Solve the equation $a - 15 = 25$ and state which axiom do you use here.
5. Ram and Ravi have the same weight. If they each gain weight by 2kg, how will their new weights be compared?
6. If a point C be the mid-point of a line segment AB, then write the relation among AC, BC and AB.
7. If a point P be the mid-point of MN and C is the mid-point of MP, then write the relation between MC and MN.
8. How many lines does pass through two distinct points?

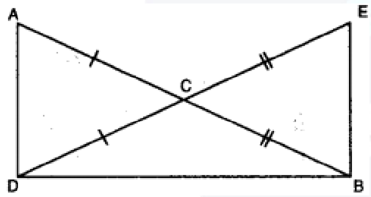
9. In the given figure, if AB = CD, then prove that AC = BD. Also, write the Euclid's axiom used for proving it.

Short Questions:

1. Define:
   (a) a square (b) perpendicular line
2. In the given figure, name the following:
   (i) Four collinear points
   (ii) Five rays
   (iii) Five-line segments
   (iv) Two-pairs of non-intersecting line segments.

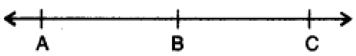


3. In the given figure, AC = DC and CB = CE. Show that AB = DE. Write the Euclid's axiom to support this

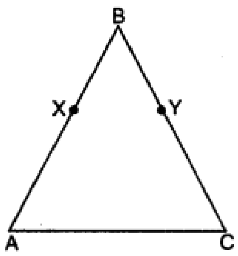


4. In figure, it is given that AD = BC. By which Euclid's axiom it can be proved that AC = BD?

5. If A, B and C are any three points on a line and B lies between A and C (see figure), then prove that AB + BC = AC

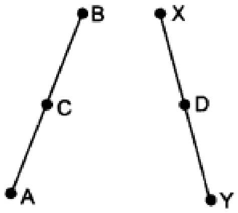


6. In the given figure, AB = BC, BX = BY, show that AX = CY.



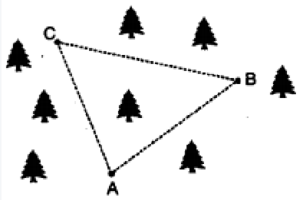
Long Questions:

1. For given four distinct points in a plane, find the number of lines that can be drawn through:



   (i) When all four points are collinear.
   (ii) When three of the four points are collinear.
   (iii) When no three of the four points are collinear.
2. Show that: length AH > sum of lengths of AB + BC + CD.

3. Rohan's maid has two children of same age. Both of them have equal number of dresses. Rohan on his birthday plans to give both of them same number of dresses. What can you say about the number of dresses each one of them will have after Rohan's birthday? Which Euclid's axiom is used to answer this question? What value is Rohan depicting by doing so? Write one more Euclid's axiom.t:
4. Three lighthouse towers are located at points A, B and C on the section of a national forest to protect animals from hunters by the forest department as shown in figure. Which value is department exhibiting by locating extra towers? How many straight lines can be drawn from A to C? State the Euclid Axiom which states the required result. Give one more. Postulate.



Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: There can be infinite number of lines that can be drawn through a single point.

Reason: From this point we can draw only two lines.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: Through two distinct points there can be only one line that can be drawn.

Reason: From this two point we can draw only one line.

Answer Key

Multiple Choice Questions

1. (c) A terminated line can be produced indefinitely on both the sides
2. (c) Three
3. (d) Zero
4. (d) Any polygon
5. (a) Surfaces
6. (b) Length and breadth only
7. (c) Lines
8. (d) The whole is lesser than the part.
9. (a) Radius
10. (c) Five

Very Short Answer:

1. Two coplanar lines (in a plane) which are not intersecting are called parallel lines. The other term intersecting is undefined.
2. Two coplanar (in a plane) lines are perpendicular if the angle between them at the point of intersection is one right angle. The other terms point of intersection and one right angle are undefined.
3. A line segment PQ of a line 'l is the continuous part of the line I with end points P and Q.

Here, continuous part of the line 'l is undefined.

4. a – 15 = 25

On adding 15 to both sides, we obtain

a–15+15 = 25+15 [using Euclid's second axiom]

a = 40

5. Let x kg be the weight each of Ram and Ravi.

On adding 2kg,

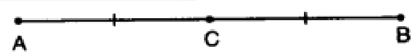
Weight of Ram and Ravi will be (x + 2) kg each.

According to Euclid's second axiom, when equals are added to equals, the wholes are equal.

6. Here, C is the mid-point of AB

⇒ AC = BC

$$\Rightarrow AC = BC = \frac{1}{2}AB$$



Hence, a = 2

7. Here, P is the mid-point of MN and C is the mid-point of MP.

$$\therefore MC = \frac{1}{4}MN$$

8. One and only one.

Here, given that AB = CD

By using Euclid's axiom 2, if equals are added to equals, then the wholes are equal, we have

AB + BC = CD + BC

⇒ AC = BD

Short Answer:

1. **Solution:**

(a) A square: A square is a rectangle having same length and breadth. Here, undefined terms are length, breadth, and rectangle.

(b) Perpendicular lines: Two coplanar (in a plane) lines are perpendicular, if the angle between them at the point of intersection is one right angle. Here, the term one right angle is undefined.

2. **Solution:**

(i) Four collinear points are D, E, F, G and H, I, J, K

(ii) Five rays are DG, EG, FG, HK, IK.

(iii) Five-line segments are DH, EI, FJ, DG, HK.

(iv) Two-pairs of non-intersecting line segments are (DH, EI) and (DG, HK).

3. **Solution:**

We have

AC = DC

CB = CE

By using Euclid's axiom 2, if equals are added to equals, then wholes are equal.

⇒ AC + CB = DC + CE

⇒ AB = DE.

4. **Solution:**

We can prove it by Euclid's axiom 3. "If equals are subtracted from equals, the remainders are equal."

We have AD = BC

⇒ AD – CD = BC – CD

⇒ AC = BD

5. **Solution:**

In the given figure, AC coincides with AB + BC. Also, Euclid's axiom 4, states that things which coincide with one another are equal to one another. So, it is evident that:

AB + BC = AC.

6. **Solution:**

Given that AB = BC

and BX = BY

By using Euclid's axiom 3, equals subtracted from equals, then the remainders are equal, we have

AB – BX = BC – BY

AX = CY

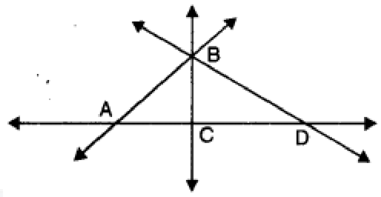
Long Answer:

1. **Solution:**

(i) Consider the points given are A, B, C and D.

When all the four points are collinear:

One line $\overrightarrow{AD}$.



(ii) When three of the four points are collinear:

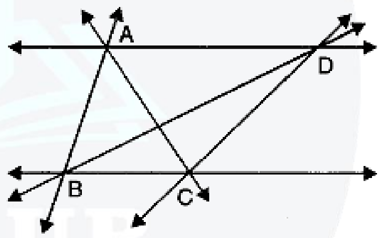
4 lines

Here, we have four lines $\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{BD}, \overleftrightarrow{CD}$ (four)

(iii) When no three of the four points are collinear:

6 lines Here, we have

$\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{AC}, \overleftrightarrow{AD}, \overleftrightarrow{BD}, \overleftrightarrow{CD}$ (six)



2. **Solution:**

We have

AH = AB + BC + CD + DE + EF + FG + GH

Clearly, AB + BC + CD is a part of AH.

⇒ AH > AB + BC + CD

Hence, length AH > sum of lengths AB + BC + CD.

3. **Solution:**

Here, Rohan's maid has two children of same age group and both of them have equal number of dresses. Rohan on his birthday plans to give both of them same number of dresses.

∴ By using Euclid's Axiom 2, if equals are added to equals, then the whole are equal. Thus, again both of them have equal number of dresses. Value depicted by Rohan are caring and other social values. According to Euclid's Axiom 3, if equals are subtracted from equals, then the remainders are equal.

4. **Solution:**

One and only one line can be drawn from A to C. According to Euclid's Postulate, "A straight line may be drawn from any point to any other point:" Another postulate: "A circle may be described with any Centre and any radius." Wildlife is a part of our environment and conservation of each of its element is important for ecological balance.

Assertion and Reason Answers-

1. (c) Assertion is correct statement but reason is wrong statement.
2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

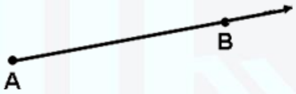
Lines and Angles | 6

Introduction to line and the terms related to it

- A **line** is a breadthless length which has no end point. Here, AB is a line and it is denoted by $\overleftrightarrow{AB}$.

- A **line segment** is a part of a line which has two end points. Here, AB is a line segment and it is denoted by $\overline{AB}$.

- A **ray** is a part of a line which has only one end point. Here, AB is a ray and it is denoted by $\overrightarrow{AB}$.

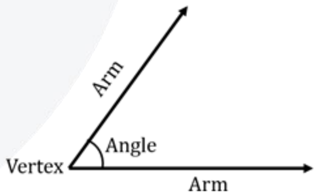


Collinear/Non-collinear points

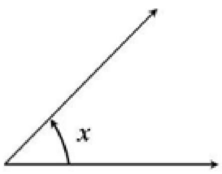
- Three or more points which lie on the same line are called **collinear points**.
- Three or more points which do not lie on a straight line are called **non-collinear points**.

Introduction to Angle

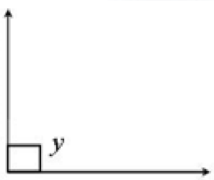
- An **angle** is formed when two rays originate from the same end point.
- The rays making an angle are called the **arms** of the angle.
- The end point from where the two rays originate to form an angle is called the **vertex** of the angle.



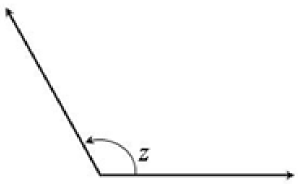
Types of angles:



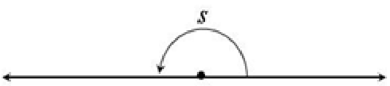
(i) Acute angle: $0° < x < 90°$



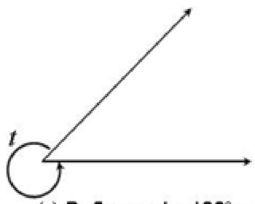
(ii) Right angle: $y = 90°$



(iii) Obtuse angle: $90° < z < 180°$



(iv) Straight angle: $s = 180°$

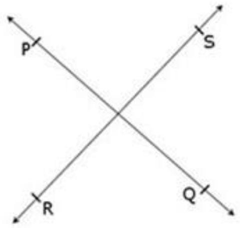


(v) Reflex angle: $180° < t < 360°$

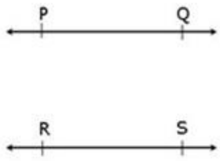
Pair of Angles

- Two angles whose sum is 90° are called **complementary angles.**
- Two angles whose sum is 180° are called **supplementary angles.**

Intersecting and non-intersecting lines



(i) Intersecting lines

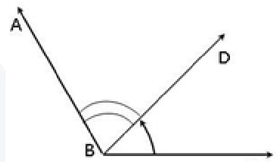


(ii) Non–intersecting (parallel) lines

Adjacent angles

Two angles are **adjacent**, if they have a common vertex, a common arm and their non–common arms are on different sides of the common arm.

In the figure, ∠ABD and ∠DBC are adjacent angles.

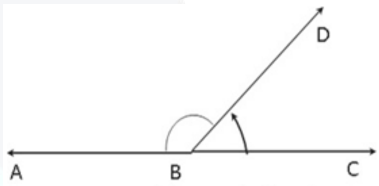


Adjacent angles

Linear pair of angles

If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and vice-versa. This property is called as the **linear pair axiom** and the angles are called **linear pair of angles.**

In the figure, ∠ABD and ∠DBC are linear pair of angles i.e. ∠ABD+∠DBC=180°.

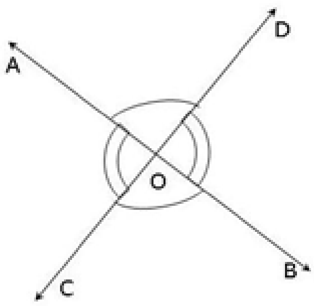


Linear pair of angles

If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

Vertically opposite angles

- The **vertically opposite angles** formed when two lines intersect each other.
- There are two pairs of vertically opposite angles in the given figure and they are ∠AOD and ∠BOC, ∠AOC and ∠BOD.



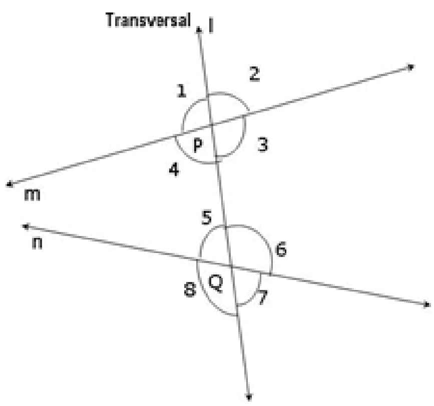
If two lines intersect each other, then the **vertically opposite angles are equal.**

Transversal

A line which intersects two or more lines at distinct points is called a **transversal.**

Pair of angles when a transversal intersects two lines

- **Corresponding angles:**
  (a) ∠1 and ∠5
  (b) ∠2 and ∠6
  (c) ∠4 and ∠8
  (d) ∠3 and ∠7
- **Alternate interior angles:**
  (a) ∠4 and ∠6
  (b) ∠3 and ∠5



- **Alternate exterior angles:**
  (a) ∠1 and ∠7
  (b) ∠2 and ∠8
- Interior angles on the same side of the transversal are referred as co-interior angles/ allied angles/ consecutive interior angles and they are:
  (a) ∠4 and ∠5
  (b) ∠3 and ∠6

If a transversal intersects two parallel lines, then

- Each pair of **corresponding angles are equal**.
- Each pair of **alternate interior angles are equal**.
- Each pair of interior angles on the same side of the transversal are supplementary.

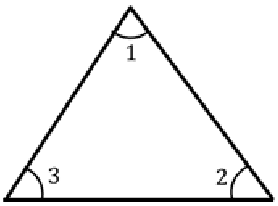
If a transversal intersects two lines

- Such that a pair of **corresponding angles** is equal, then the two **lines are parallel.**
- Such that a pair of **alternate interior angles** is equal, then the two **lines are parallel**.
- Such that a pair of **interior angles** on the same side of the transversal is supplementary, then the two **lines are parallel**.
- Such that the bisectors of a pair of **corresponding angles** are parallel, then the two **lines are parallel.**

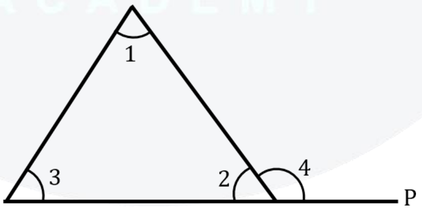
Lines parallel to the same line

Two lines which are parallel to the same line are parallel to each other. This holds for more than two lines also i.e. if two or more lines are parallel to the same line then they will be parallel to each other.

Angle sum property of a triangle

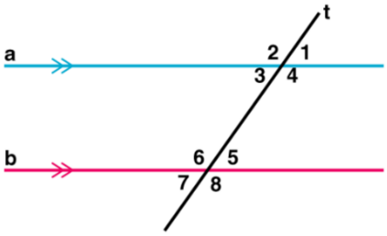


- The sum of the angles of a triangle is 180°. This is known as the **angle sum property of a triangle**.
  Here, ∠1+∠2+∠3=180°.
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles. This is known as the **exterior angle property of a triangle**.



  Here, ∠4=∠1+∠3.
- An exterior angle of a triangle is greater than either of its interior opposite angles. In the above figure, ∠4>∠1 and ∠4>∠3.

Parallel lines with a transversal



- ∠1 = ∠5, ∠2 = ∠6, ∠4 = ∠8 and ∠3 = ∠7 (Corresponding angles)
- ∠3 = ∠5, ∠4 = ∠6 (Alternate interior angles)
- ∠1 = ∠7, ∠2 = ∠8 (Alternate exterior angles)

Geometry Symbol Chart

When 2 rays originate from the same point at different directions, they form an angle.
The rays are called arms and the common point is called the vertex

Types of angles:

- Acute angle 0°< a < 90°
- Right angle a = 90°
- Obtuse angle: 90° < a < 180°
- Straight angle =180°
- Reflex Angle 180° < a < 360°
- Angles that add up to 90° are complementary angles
- Angles that add up to 180° are called supplementary angles. The symbols for angles and triangles are most important and frequently used symbols in geometry.

Intersecting Lines and Associated Angles

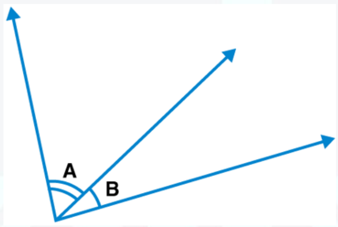
Intersecting and Non-Intersecting lines

When 2 lines meet at a point they are called intersecting

When 2 lines never meet at a point, they are called non-intersecting or parallel lines

Adjacent angles

2 angles are adjacent if they have the same vertex and one common point.



Linear Pair

When 2 adjacent angles are supplementary, i.e they form a straight line (add up to 180∘), they are called a linear pair.

Vertically opposite angles

When two lines intersect at a point, they form equal angles that are vertically opposite to each other.

Basic Properties of a Triangle

All the properties of a triangle are based on its sides and angles. By the definition of triangle, we know that it is a closed polygon that consists of three sides and three vertices. Also, the sum of all three internal angles of a triangle equal to 180°.

Depending upon the length of sides and measure of angles, the triangles are classified into different types of triangles.

In the beginning, we start from understanding the shape of triangles, its types and properties, theorems based on it such as Pythagoras theorem, etc. In higher classes, we deal with trigonometry, where the right-angled triangle is the base of the concept. Let us learn here some of the fundamentals of the triangle by knowing its properties.

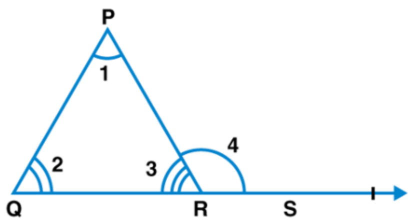
Triangle and sum of its internal angles

Sum of all angles of a triangle add up to 180°

An exterior angle of a triangle = sum of opposite internal angles

– If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles

– $\angle 4 = \angle 1 + \angle 2$



Types of Triangle

Coordinates are the two ordered pair, which defines the location of any given point in a plane. Let's understand it with the help of the box below.

Based on the Sides	Based on the Angles
Scalene Triangle	Acute angled Triangle
Isosceles Triangle	Right angle Triangle
Equilateral Triangle	Obtuse-angled Triangle

So before, discussing the properties of triangles, let us discuss types of triangles given above.

Scalene Triangle: All the sides and angles are unequal.

Isosceles Triangle: It has two equal sides. Also, the angles opposite these equal sides are equal.

Equilateral Triangle: All the sides are equal and all the three angles equal to 60°.

Acute Angled Triangle: A triangle having all its angles less than 90°.

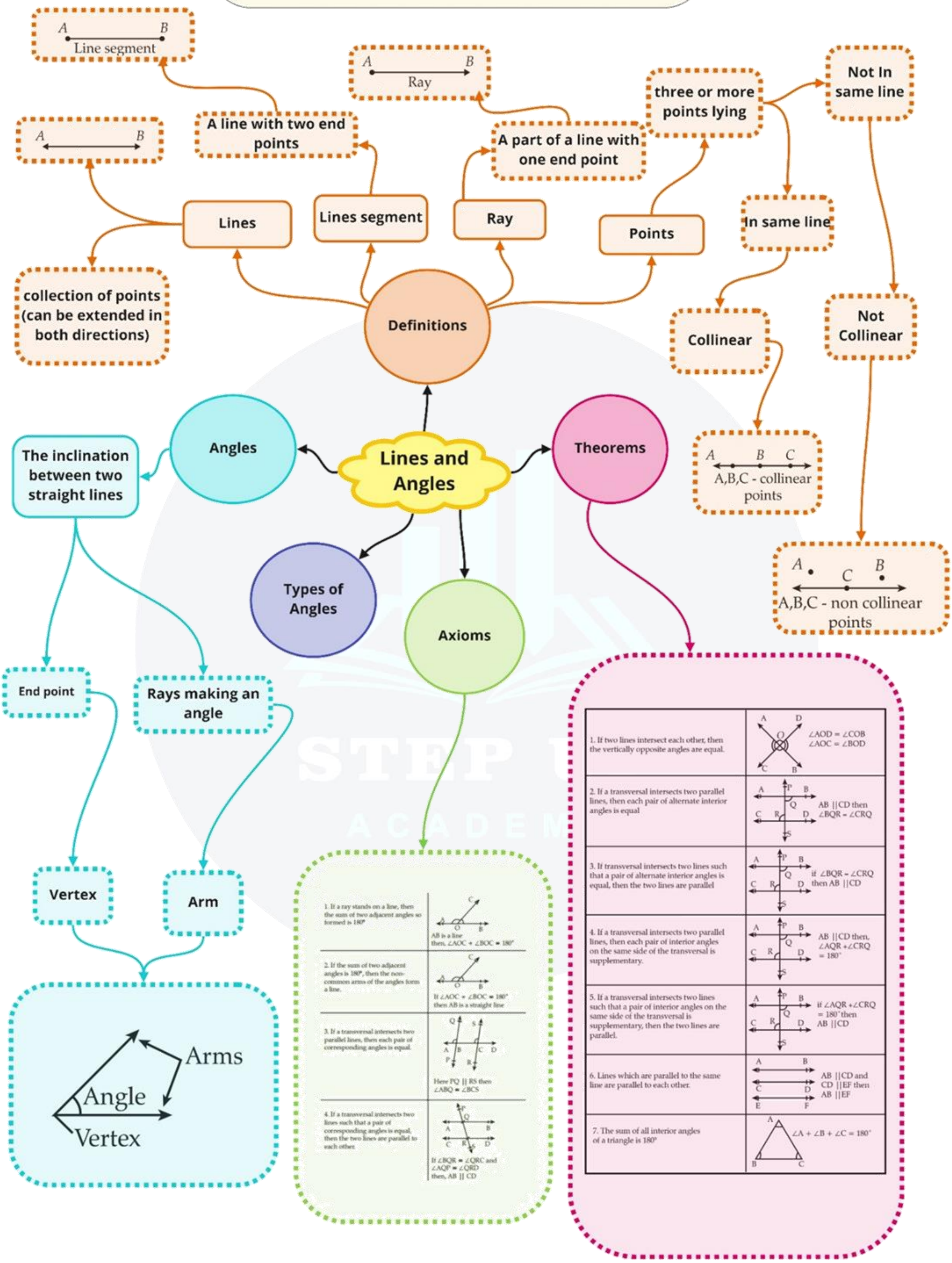
Right Angled Triangle: A triangle having one of the three angles exactly 90°.

Obtuse Angled Triangle: A triangle having one of the three angles more than 90°.

Triangle Formula

- Area of a triangle is the region occupied by a triangle in a two-dimensional plane. The dimension of the area is square units. The formula for area is given by;
  Area = 1/2 x Base x Height
- The perimeter of a triangle is the length of the outer boundary of a triangle. To find the perimeter of a triangle we need to add the length of the sides of the triangle.
  P = a + b + c
- Semi-perimeter of a triangle is half of the perimeter of the triangle. It is represented by s.
  s = (a + b + c)/2
  where a, b and c are the sides of the triangle.
- By Heron's formula, the area of the triangle is given by:
  $A = \sqrt{[s(s-a)(s-b)(s-c)]}$
  where 's' is the semi-perimeter of the triangle.
- By the Pythagorean theorem, the hypotenuse of a right-angled triangle can be calculated by the formula:
  $\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$

Class : 9th mathematics
Chapter- 6: Lines and Angles



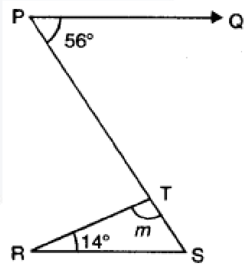
Important Questions

Multiple Choice Questions

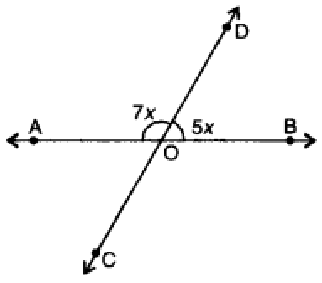
1. In a right-angled triangle where angle A = 90° and AB = AC. What are the values of angle B?
   (a) 45°
   (b) 35°
   (c) 75°
   (d) 65°
2. In a triangle ABC if ∠A = 53° and ∠C = 44° then the value of ∠B is:
   (a) 46°
   (b) 83°
   (c) 93°
   (d) 73°
3. Given four points such that no three of them are collinear, then the number of lines that can be drawn through them are:
   (a) 4 lines
   (b) 8 lines
   (c) 6 lines
   (d) 2 lines
4. If one angle of triangle is equal to the sum of the other two angles then triangle is:
   (a) Acute triangle
   (b) Obtuse triangle
   (c) Right triangle
   (d) None of these
5. How many degrees are there in an angle which equals one-fifth of its supplement?
   (a) 15°
   (b) 30°
   (c) 75°
   (d) 150°
6. Sum of the measure of an angle and its vertically opposite angle is always.
   (a) Zero
   (b) Thrice the measure of the original angle
   (c) Double the measure of the original angle
   (d) Equal to the measure of the original angle
7. If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.
   (a) Equal
   (b) Complementary
   (c) Supplementary
   (d) Corresponding
8. The bisectors of the base angles of an isosceles triangle ABC, with AB = AC, meet at O. If ∠B = ∠C = 50Â°. What is the measure of angle O?
   (a) 120°
   (b) 130°
   (c) 80°
   (d) 150°
9. The angles of a triangle are in the ratio 2 : 3 : 4. The angles, in order, are :
   (a) 80°, 40°, 60°
   (b) 20°, 60°, 80°
   (c) 40°, 60°, 80°
   (d) 60°, 40°, 80°
10. An acute angle is:
    (a) More than 90 degrees
    (b) Less than 90 degrees
    (c) Equal to 90 degrees
    (d) Equal to 180 degrees

Very Short Questions:

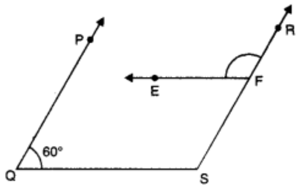
1. If an angle is half of its complementary angle, then find its degree measure.
2. The two complementary angles are in the ratio 1 : 5. Find the measures of the angles.
3. In the given figure, if PQ || RS, then find the measure of angle m.



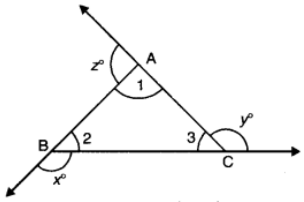
4. If an angle is 14° more than its complement, then find its measure.
5. If AB || EF and EF || CD, then find the value of x.
6. In the given figure, lines AB and CD intersect at O. Find the value of x.



7. In the given figure, PQ || RS and EF || QS. If ∠PQS = 60°, then find the measure of ∠RFE.

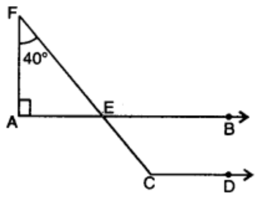


8. In the given figure, if x°, y° and z° are exterior angles of ΔABC, then find the value of x° + y° + z°.

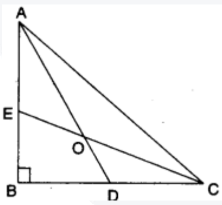


Short Questions:

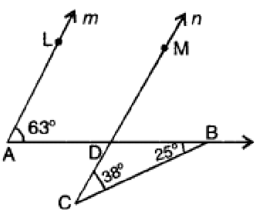
1. In the given figure, AB || CD, ∠FAE=90°, ∠AFE=40°, find ∠ECD.



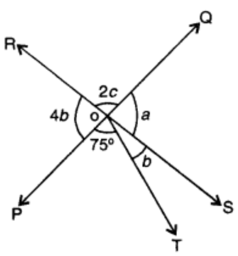
2. In the fig., AD and CE are the angle bisectors of ∠A and ∠C respectively. If ∠ABC = 90°, then find ∠AOC.



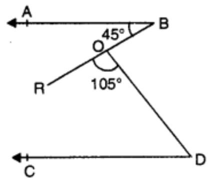
3. In the given figure, prove that m || n.



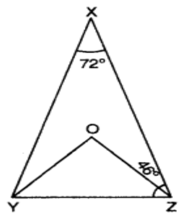
4. In the given figure, two straight lines PQ and RS intersect each other at O. If ∠POT = 75°, find the values of a, b, c.



5. In figure, if AB || CD. If ∠ABR = 45° and ∠ROD = 105°, then find ∠ODC.

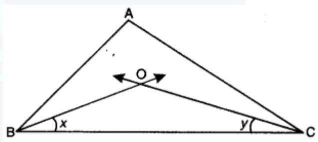


6. In the figure, ∠X = 72°, ∠XZY = 46°. If YO and ZO are bisectors of ∠XYZ and ∠XZY respectively of ΔXYZ, find ∠OYZ and ∠YOZ.

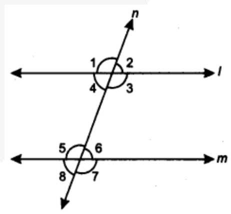


Long Questions:

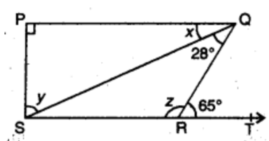
1. If two parallel lines are intersected by a transversal, prove that the bisectors of two pairs of interior angles form a rectangle.
2. If in ΔABC, the bisectors of ∠B and ∠C intersect each other at O. Prove that ∠BOC = $90^\circ \frac{1}{2} < A$



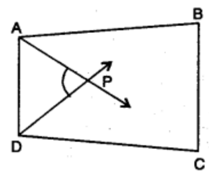
3. In figure, if l || m and ∠1 = (2x + y)°, ∠4 = (x + 2y)° and ∠6 = (3y + 20)°. Find ∠7 and ∠8.



4. In the given figure, if PQ ⊥ PS, PQ || SR, ∠SQR = 28o and ∠QRT = 65°. Find the values of x, y and z.



5. In figure, AP and DP are bisectors of two adjacent angles A and D of a quadrilateral ABCD. Prove that 2∠APD = ∠B + ∠C.



Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
    a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
    b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
    c) Assertion is correct statement but reason is wrong statement.
    d) Assertion is wrong statement but reason is correct statement.

    Assertion: Two adjacent angles always form a linear pair.

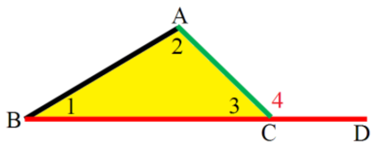
    Reason: In a linear pair of angles two non-common arms are opposite rays.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
    a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
    b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
    c) Assertion is correct statement but reason is wrong statement.
    d) Assertion is wrong statement but reason is correct statement.

    Assertion: A triangle can have tow obtuse angles.

    Reason: Sum of the three angles in a triangle is always 180°.

Case Study Questions:

1. Read the Source/ Text given below and answer these questions:

    Ashok is studying in 9th class in Govt School, Chhatarpur. Once he was at his home and was doing his geometry homework. He was trying to measure three angles of a triangle using the Dee, but his dee was old and his Dee's numbers were erased and the lines on the dee were visible. Let us help Ashok to find the angles of the triangle. He found that the second angle of the triangle was three times as large as the first. The measure of the third angle is double of the first angle.

    Now answer the following questions:

    i. What was the value of the first angle?
        a. 30°
        b. 45°
        c. 60°
        d. 90°

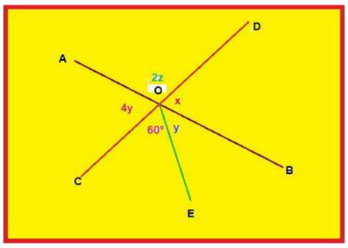
    ii. What was the value of the third angle?
        a. 30°
        b. 45°
        c. 60°
        d. 90°

    iii. What was the value of the second angle?
        a. 30°
        b. 45°
        c. 60°
        d. 90°

    iv. Which is the solution satisfying both the equations formed in (iii)?
        a. 120°
        b. 45°
        c. 60°
        d. 90°

    v. What was the sum of all three angles measured by Ashok using Dee?
        a. 270°
        b. 180°
        c. 100°
        d. 90°

2. Read the Source/ Text given below and answer any four questions:

    Maths teacher draws a straight line AB shown on the blackboard as per the following figure.

    i. Now he told Raju to draw another line CD as in the figure.

    ii. The teacher told Ajay to mark ∠AOD as 2z.

iii. Suraj was told to mark ∠AOC as 4y.
iv. Clive Made and angle ∠COE=60∘.
v. Peter marked ∠BOE and ∠BOD as y and x respectively.

Now answer the following questions:

i. What is the value of x?
  a. 48°
  b. 96°
  c. 100°
  d. 120°

ii. What is the value of y?
  a. 48°
  b. 96°
  c. 100°
  d. 24°

iii. What is the value of z?
  a. 48°
  b. 96°
  c. 42°
  d. 120°

iv. What should be the value of x + 2z?
  a. 148°
  b. 360°
  c. 180°
  d. 120

v. What is the relation between y and z?
  a. 2y + z = 90°
  b. 2y + z = 180°
  c. 4y + 2z = 120°
  d. y = 2z

Answer Key

Multiple Choice Questions

1. (b) 45°
2. (b) 83°
3. (c) 6 lines
4. (c) Right triangle
5. (b) 30°
6. (c) Double the measure of the original angle
7. (d) Corresponding
8. (b) 130°
9. (c) 40°, 60°, 80°
10. (b) Less than 90 degrees

Very Short Answer:

1. Let the required angle be x

∴ Its complement = 90° – x

Now, according to given statement, we obtain

$x = \frac{1}{2}(90° - x)$

⇒ 2x = 90° – x

⇒ 3x = 90°

⇒ x = 30°

Hence, the required angle is 30°.

2. Let the two complementary angles be x and 5x.

∴ x + 5x = 90°

⇒ 6x = 90°

⇒ x = 15°

3. Here, PQ || RS, PS is a transversal.

⇒ ∠PSR = ∠SPQ = 56°

Also, ∠TRS + m + ∠TSR = 180°

14° + m + 56° = 180°

⇒ m = 180° – 14 – 56 = 110°

4. Let the required angle be x

∴ Its complement = 90° – x

Now, according to given statement, we obtain

x = 90° – x + 14°

⇒ 2x = 104°

⇒ x = 52°

Hence, the required angle is 52°.

5. Since EF || CD ∴ y + 150° = 180°

⇒ y = 180° – 150° = 30°

Now, ∠BCD = ∠ABC

x + y = 70°

x + 30 = 70

⇒ x = 70° – 30° = 40°

Hence, the value of x is 40°.

6. Here, lines AB and CD intersect at O.

∴ ∠AOD and ∠BOD forming a linear pair

⇒ ∠AOD + ∠BOD = 180°

⇒ 7x + 5x = 180°

⇒ 12x = 180°

⇒ x = 15°

7. Since PQ || RS

$\therefore \angle PQS + \angle QSR = 180°$

$\Rightarrow 60° + \angle QSR = 180°$

$\Rightarrow \angle QSR = 120°$

Now, EF || QS

$\Rightarrow \angle RFE = \angle QSR$ [corresponding ∠s]

$\Rightarrow \angle RFE = 120°$

8. We know that an exterior angle of a triangle is equal to sum of two opposite interior angles.

$\Rightarrow x° = \angle 1 + \angle 3$

$\Rightarrow y° = \angle 2 + \angle 1$

$\Rightarrow z° = \angle 3 + \angle 2$

Adding all these, we have

$x° + y° + z° = 2(\angle 1 + \angle 2 + \angle 3)$

$= 2 \times 180°$

$= 360°$

Short Answer:

1. **Solution:**

In AFAE,

ext. $\angle FEB = \angle A + F$

$= 90° + 40° = 130°$

Since AB || CD

$\therefore \angle ECD = FEB = 130°$

Hence, $\angle ECD = 130°$.

2. **Solution:**

$\because$ AD and CE are the bisector of ∠A and ∠C

$\therefore \quad \angle OAC = \frac{1}{2}\angle A$ and $\angle OCA = \frac{1}{2}\angle C$

$\Rightarrow \angle OAC + \angle OCA = \frac{1}{2}(\angle A + \angle C)$

$= \frac{1}{2}(180° - \angle B) \qquad [\because \angle A + \angle B + \angle C = 180°]$

$= \frac{1}{2}(180° - 90°) \qquad [\because \angle ABC = 90°]$

$= \frac{1}{2} \times 90° = 45°$

In ΔAOC,

$\angle AOC + \angle OAC + \angle OCA = 180°$

$\Rightarrow \angle AOC + 45° = 180°$

$\Rightarrow \angle AOC = 180° - 45° = 135°$

3. **Solution:**

In ΔBCD,

ext. $\angle BDM = \angle C + \angle B$

$= 38° + 25° = 63°$

Now, $\angle LAD = \angle MDB = 63°$

But these are corresponding angles.

Hence, m || n

4. **Solution:**

Here, $4b + 75° + b = 180°$ [a straight angle]

$5b = 180° - 75° = 105°$

$b - \frac{105°}{5} = 21°$

$\therefore a = 4b = 4 \times 21° = 84°$ (vertically opp. ∠s)

Again, $2c + a = 180°$ [a linear pair]

$\Rightarrow 2c + 84° = 180°$

$\Rightarrow 2c = 96°$

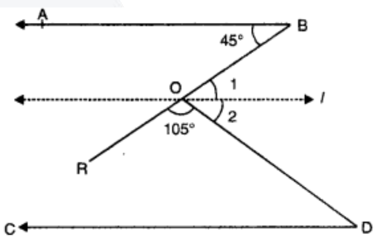
$\Rightarrow c = \frac{96°}{2} = 48°$

Hence, the values of a, b and c are a = 84°, b = 21° and c = 48°.

5. **Solution:**

Through O, draw a line 'l' parallel to AB.

$\Rightarrow$ line I will also parallel to CD, then



$\angle 1 = 45°$ [alternate int. angles]

$\angle 1 + \angle 2 + 105° = 180°$ [straight angle]

$\angle 2 = 180° - 105° - 45°$

$\Rightarrow \angle 2 = 30°$

Now, $\angle ODC = \angle 2$ [alternate int. angles]

$= \angle ODC = 30°$

6. **Solution:**

In ΔXYZ, we have

$\angle X + XY + \angle Z = 180°$

$\Rightarrow \angle Y + \angle Z = 180° - \angle X$

$\Rightarrow \angle Y + \angle Z = 180° - 72°$

$\Rightarrow Y + \angle Z = 108°$

$\angle OYZ + \angle OZY = 54°$

[$\because$ YO and ZO are the bisector of ∠XYZ and ∠XZY]

$\Rightarrow \angle OYZ + \frac{1}{2} \times 46° = 54°$

$\angle OYZ + 23° = 54°$

$\Rightarrow \angle OYZ = 549 - 23° = 31°$

In ΔYOZ, we have

$\angle YOZ = 180° - (\angle OYZ + \angle OZY)$

$= 180° - (31° + 23°)\ 180° - 54° = 126°$

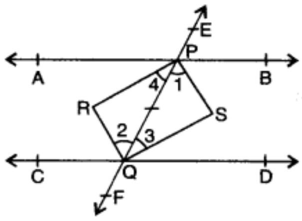
Long Answer:

1. **Solution:**

Given: AB || CD and transversal EF cut them at P and Q respectively and the bisectors of pair of interior angles form a quadrilateral PRQS.

To Prove: PRQS is a rectangle.

Proof: ∵ PS, QR, QS and PR are the bisectors of angles ∠BPQ, ∠CQP, ∠DQP and ∠APQ respectively.



$\therefore \angle 1 = \frac{1}{2}\angle BPQ, \angle 2 = \frac{1}{2}\angle CPQ,$

$\angle 3 = \frac{1}{2}\angle DQP$ and $\angle 4 = \frac{1}{2}\angle APQ$

Now, AB || CD and EF is a transversal

∴ ∠BPQ = ∠CQP

$\Rightarrow \angle 1 = \angle 2$ ($\because \angle 1 = \frac{1}{2}\angle BPQ$ and $\angle 2 = \frac{1}{2}\angle CPQ$)

But these are pairs of alternate interior angles of PS and QR

∴ PS || QR

Similarly, we can prove ∠3 = ∠4 = QS || PR

∴ PRQS is a parallelogram.

Further $\angle 1 + \angle 3 = \frac{1}{2}\angle BPQ + \frac{1}{2}\angle DQP$

$= \frac{1}{2}(\angle BPQ + \angle DQP)$

$= \frac{1}{2} \times 180° = 90°$ ($\because \angle BPQ + \angle DQP = 180°$)

∴ In ΔPSQ, we have ∠PSQ = 180° – (∠1 + ∠3)

= 180° – 90° = 90°

Thus, PRRS is a parallelogram whose one angle ∠PSQ = 90°.

Hence, PRQS is a rectangle.

2. **Solution:**

Let ∠B = 2x and ∠C = 2y

∵ OB and OC bisect ∠B and ∠C respectively.

$\angle OBC = \frac{1}{2}\angle B = \frac{1}{2} \times 2x = x$

and $\angle OBC = \frac{1}{2}\angle C = \frac{1}{2} \times 2y = y$

Now, in ΔBOC, we have

∠BOC + ∠OBC + ∠OBC = 180°

⇒ ∠BOC + x + y = 180°

⇒ ∠BOC = 180° – (x + y) ....(i)

Now, in ΔABC, we have

∠A + 2B + C = 180°

⇒ ∠A + 2x + 2y = 180°

$\Rightarrow 2(x + y) = \frac{1}{2}(180° - \angle A)$

$\Rightarrow x + y = 90° - \frac{1}{2}\angle A$ ....(ii)

From (i) and (ii), we have

$\angle BOC = 180° - (90° - \frac{1}{2}\angle A) = 90° + \frac{1}{2}\angle A$

3. **Solution:**

Here, ∠1 and ∠4 are forming a linear pair

∠1 + ∠4 = 180°

(2x + y)° + (x + 2y)° = 180°

3(x + y)° = 180°

x + y = 60

Since I || m and n is a transversal

∠4 = ∠6

(x + 2y)° = (3y + 20)°

x – y = 20

Adding (i) and (ii), we have

2x = 80 = x = 40

From (i), we have

40 + y = 60 ⇒ y = 20

Now, ∠1 = (2 x 40 + 20)° = 100°

∠4 = (40 + 2 x 20)° = 80°

∠8 = ∠4 = 80° [corresponding ∠s]

∠1 = ∠3 = 100° [vertically opp. ∠s]

∠7 = ∠3 = 100° [corresponding ∠s]

Hence, ∠7 = 100° and ∠8 = 80°

4. **Solution:**

Here, PQ || SR.

⇒ ∠PQR = ∠QRT

⇒ x + 28° = 65°

⇒ x = 65° – 28° = 37°

Now, in it. ΔSPQ, ∠P = 90°

∴ ∠P + x + y = 180° [angle sum property]

∴ 90° + 37° + y = 180°

⇒ y = 180° – 90° - 37° = 53°

Now, ∠SRQ + ∠QRT = 180° [linear pair]

z + 65° = 180°

z = 180° - 65° = 115°

5. **Solution:**

In quadrilateral ABCD, we have

$\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C + \frac{1}{2}\angle D = \frac{1}{2} \times 360°$

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle D = 180° - \frac{1}{2}(\angle B + \angle C)$

As, AP and DP are the bisectors of ∠A and ∠D

$\therefore \quad \angle PAD = \frac{1}{2}\angle A$

and $\quad \angle PDA = \frac{1}{2}\angle D$

Now, $\angle PAD + \angle PDA = 180° - \frac{1}{2}(\angle B + \angle C)$ ....(i)

In ΔAPD, we have

$\angle APD + \angle PAD + \angle PDA = 180°$

$\Rightarrow \angle APD + 180° - \frac{1}{2}(\angle B + \angle C) = 180°$ [using (i)]

$\Rightarrow \quad \angle APD = \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow \quad 2\angle APD = \angle B + \angle C$

Assertion and Reason Answers-

1. (d) Assertion is wrong statement but reason is correct statement.

Explanation:

Linear pair

Adjacent angles with opposite rays as noncommon arms are called the linear pair.

They form a straight angle.

Hence **Reason is True.**

Two adjacent angles form a linear pair if non common arms are opposite rays.

If non common sides are not opposite rays then adjacent angles does not form a linear pair.

Hence Assertion "Two adjacent angles always form a linear pair" is False

For example two adjacent angles which are complementary forms a right angle not a linear pair.

2. (d) Assertion is wrong statement but reason is correct statement.

Explanation:

ASSERTION : A triangle can have two obtuse angles.

Obtuse angle are the angles whose measure are between 90° and 180°

If a triangle has two obtuse angles then sum of those two angles will be between (90° + 90°) and (180° + 180°) = between 180° and 360°.

Hence sum of all the angles of triangle would be greater than 180°.

But Sum of all the angles of a triangle is 180°

Hence This is not possible, so Assertion is FALSE

REASON : The sum of all the interior angles of a triangle is 180°.

TRUE

Case Study Answer:

1.

(i)	(a)	30°
(ii)	(c)	60°
(iii)	(d)	90°
(iv)	(a)	120°
(v)	(b)	180°

2.

(i)	(b)	96°
(ii)	(d)	24°
(iii)	(c)	42°
(iv)	(c)	180°
(v)	(a)	2y + z = 90°

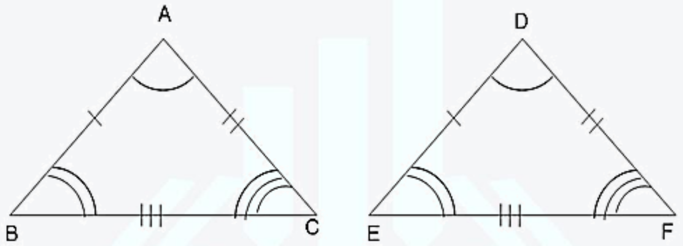
Triangles 7

What are congruent figures

- Two figures are said to be **congruent** if they are of the same shape and of the same size.
- Two circles of the same radii are congruent.
- Two squares of the same sides are congruent.

Congruent triangles

If two triangles ABC and DEF are congruent under the correspondence A ↔ D, B ↔ E and C ↔ F, then symbolically, it is expressed as $\Delta ABC \cong \Delta DEF$.

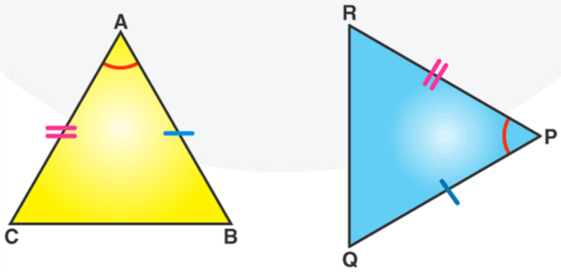


In congruent triangles, **corresponding parts are equal.** We write in short 'CPCT' for corresponding parts of congruent triangles.

SAS (Side – Angle – Side) congruence rule

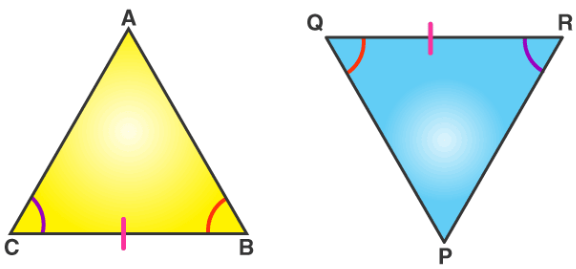
Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

Note: SAS congruence rule holds but not ASS or SSA rule.



ASA (Angle – Side – Angle) congruence rule

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.



AAS (Angle – Angle – Side) congruence rule

Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

AAS congruency can be proved in easy steps. Suppose we have two triangles ABC and DEF, where,

∠B = ∠E [Corresponding sides] ∠C = ∠F [Corresponding sides] And

AC = DF [Adjacent sides]

By angle sum property of triangle, we know that;

∠A + ∠B + ∠C = 180 ………(1)

∠D + ∠E + ∠F = 180 ………(2)

From equation 1 and 2 we can say;

∠A + ∠B + ∠C = ∠D + ∠E + ∠F

∠A + ∠E + ∠F = ∠D + ∠E + ∠F [Since, ∠B = ∠E and ∠C = ∠F] ∠A = ∠D

Hence, in triangle ABC and DEF,

∠A = ∠D

AC = DF

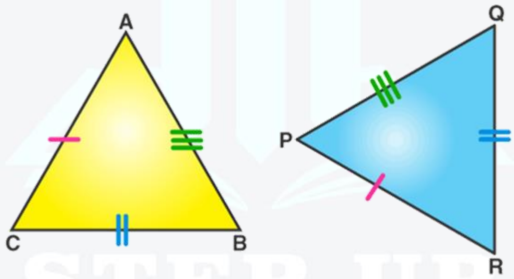
∠C = ∠F

Hence, by ASA congruency,

Δ ABC ≅ Δ DEF

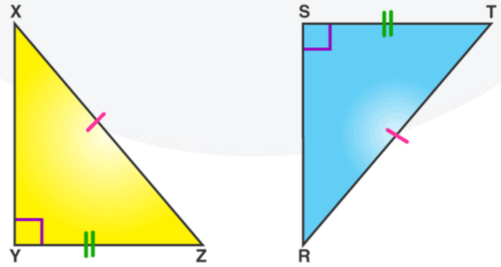
SSS (Side – Side – Side) congruent rule

If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.



RHS (Right Angle – Hypotenuse – Side) congruence rule

If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.



In above figure, hypotenuse XZ = RT and side YZ=ST, hence triangle XYZ ≅ triangle RST.

Congruence of Triangles

Congruence of triangles: Two triangles are said to be congruent if all three corresponding sides are equal and all the three corresponding angles are equal in measure. These triangles can be slides, rotated, flipped and turned to be looked identical. If repositioned, they coincide with each other. The symbol of congruence is' ≅'.

The corresponding sides and angles of congruent triangles are equal. There are basically four congruency rules that proves if two triangles are congruent. But it is necessary to find all six dimensions. Hence, the congruence of triangles can be evaluated by knowing only three values out of six. The meaning of congruence in Maths is when two figures are similar to each other based on their shape and size. Also, learn about Congruent Figures here.

Congruence is the term used to define an object and its mirror image. Two objects or shapes are said to be congruent if they superimpose on each other. Their shape and dimensions are the same. In the case of geometric figures, line segments with the same length are congruent and angles with the same measure are congruent.

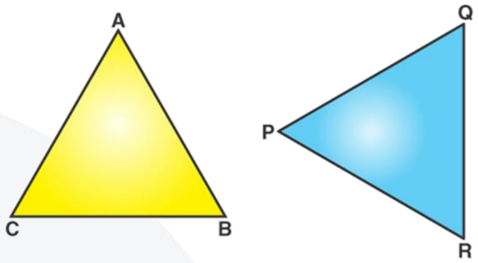
CPCT is the term, we come across when we learn about the congruent triangle. Let's see the condition for triangles to be congruent with proof.

Congruent meaning in Maths

The meaning of congruent in Maths is addressed to those figures and shapes that can be repositioned or flipped to coincide with the other shapes. These shapes can be reflected to coincide with similar shapes.

Two shapes are congruent if they have the same shape and size. We can also say if two shapes are congruent, then the mirror image of one shape is same as the other.

Congruent Triangles



A polygon made of three line segments forming three angles is known as a Triangle.

Two triangles are said to be congruent if their sides have the same length and angles have same measure. Thus, two triangles can be superimposed side to side and angle to angle.

In the above figure, Δ ABC and Δ PQR are congruent triangles. This means,

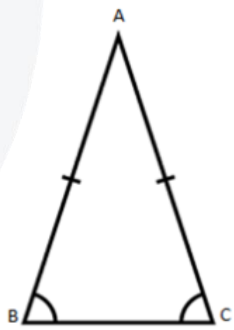
Vertices: A and P, B and Q, and C and R are the same.

Sides: AB=PQ, QR= BC and AC=PR;

Angles: ∠A = ∠P, ∠B = ∠Q, and ∠C = ∠R.

Congruent triangles are triangles having corresponding sides and angles to be equal. Congruence is denoted by the symbol "≅". They have the same area and the same perimeter.

Isosceles triangle and its properties



- A triangle in which two sides are equal is called an **isosceles** triangle.
- Angles opposite to equal sides of an isosceles triangle are equal.
- The sides opposite to equal angles of a triangle are equal.

An isosceles triangle definition states it as a polygon that consists of two equal sides, two equal angles, three edges, three vertices and the sum of internal angles of a triangle equal to 1800. In this section, we will discuss the properties of isosceles triangle along with its definitions and its significance in Maths.

Apart from the isosceles triangle, there is a different classification of triangles depending upon the sides and angles, which have their own individual properties as well. Below is the list of types of triangles;

- Scalene Triangle
- Equilateral Triangle
- Acute angled Triangle
- Right angle Triangle
- Obtuse-angled Triangle

Isosceles triangle basically has two equal sides and angles opposite to these equal sides are also equal. Same like the Isosceles triangle, scalene and equilateral are also classified on the basis of their sides, whereas acute-angled, right-angled and obtuse-angled triangles are defined on the basis of angles. So before, discussing the properties of isosceles triangles, let us discuss first all the types of triangles.

Below are basic definitions of all types of triangles:

Scalene Triangle: A triangle which has all the sides and angles, unequal.

Equilateral Triangle: A triangle whose all the sides are equal and all the three angles are of 600.

Acute Angled Triangle: A triangle having all its angles less than right angle or 900.

Right Angled Triangle: A triangle having one of the three angles as right angle or 900.

Obtuse Angled Triangle: A triangle having one of the three angles as more than right angle or 900.

Isosceles Triangle Properties

An Isosceles Triangle has the following properties:

- Two sides are congruent to each other.
- The third side of an isosceles triangle which is unequal to the other two sides is called the base of the isosceles triangle.
- The two angles opposite to the equal sides are congruent to each other. That means it has two congruent base angles and this is called an isosceles triangle base angle theorem.
- The angle which is not congruent to the two congruent base angles is called an apex angle.
- The altitude from the apex of an isosceles triangle bisects the base into two equal parts and also bisects its apex angle into two equal angles.
- The altitude from the apex of an isosceles triangle divides the triangle into two congruent right-angled triangles.
- Area of Isosceles triangle = ½ × base × altitude
- Perimeter of Isosceles triangle = sum of all the three sides

Example: If an isosceles triangle has lengths of two equal sides as 5 cm and base as 4 cm and an altitude are drawn from the apex to the base of the triangle. Then find its area and perimeter.

Solution: Given the two equal sides are of 5 cm and base is 4 cm.

We know, the area of Isosceles triangle = ½ × base × altitude

Therefore, we have to first find out the value of altitude here.

The altitude from the apex divides the isosceles triangle into two equal right angles and bisects the base into two equal parts. Thus, by Pythagoras theorem,

Hypotenuse2 = Base2 + Perpendicular2

Or Perpendicular = $\sqrt{\text{Hypotenuse}^2 - \text{Base}^2}$

∴ Altitude = $\sqrt{5^2 - 2^2}$

= $\sqrt{25-4}$ = $\sqrt{21}$

So, the area of Isosceles triangle = ½ × 4 × $\sqrt{21}$ = $2\sqrt{21}$ cm²

Perimeter of Isosceles triangle = sum of all the sides of the triangle

= 5 + 4 + 5 cm = 14 cm

Inequalities in a triangle

- If two sides of a triangle are unequal, the angle opposite to the longer side is greater.
- In any triangle, the **side opposite to greater (larger) angle is longer**.
- The **sum of any two sides** of a triangle is **greater than the third side**.
- The **difference between any two sides** of a triangle is **less than the third side**.

Relationship between unequal sides of the triangle and the angles opposite to it.

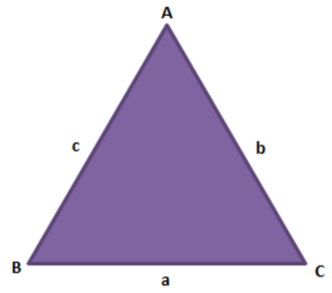
If 2 sides of a triangle are unequal, then the angle opposite to the longer side will be larger than the angle opposite to the shorter side.

Triangle inequality

The sum of the lengths of any two sides of a triangle must be greater than the third side.

According to triangle inequality theorem, for any given triangle, the sum of two sides of a triangle is always greater than the third side. A polygon bounded by three line-segments is known as the Triangle. It is the smallest possible polygon. A triangle has three sides, three vertices, and three interior angles. The types of triangles are based on its angle measure and length of the sides. The inequality theorem is applicable for all types triangles such as equilateral, isosceles and scalene. Now let us learn this theorem in details with its proof.

Triangle Inequality Theorem Proof



The triangle inequality theorem describes the relationship between the three sides of a triangle. According to this theorem, for any triangle, the sum of lengths of two sides is always greater than the third side. In other words, this theorem specifies that the shortest distance between two distinct points is always a straight line.

Consider a ΔABC as shown below, with a, b and c as the side lengths.

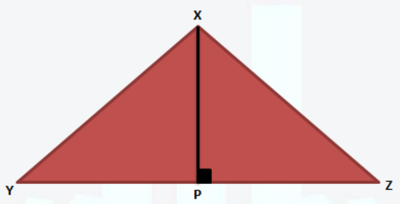
The triangle inequality theorem states that:

$a < b + c$,

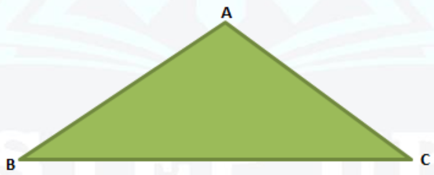
$b < a + c$,

$c < a + b$

In any triangle, the shortest distance from any vertex to the opposite side is the Perpendicular. In figure below, XP is the shortest line segment from vertex X to side YZ.

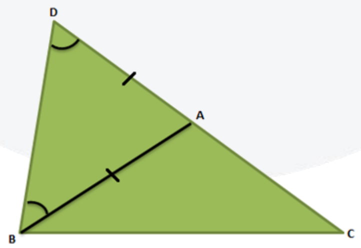


Let us prove the theorem now for a triangle ABC.



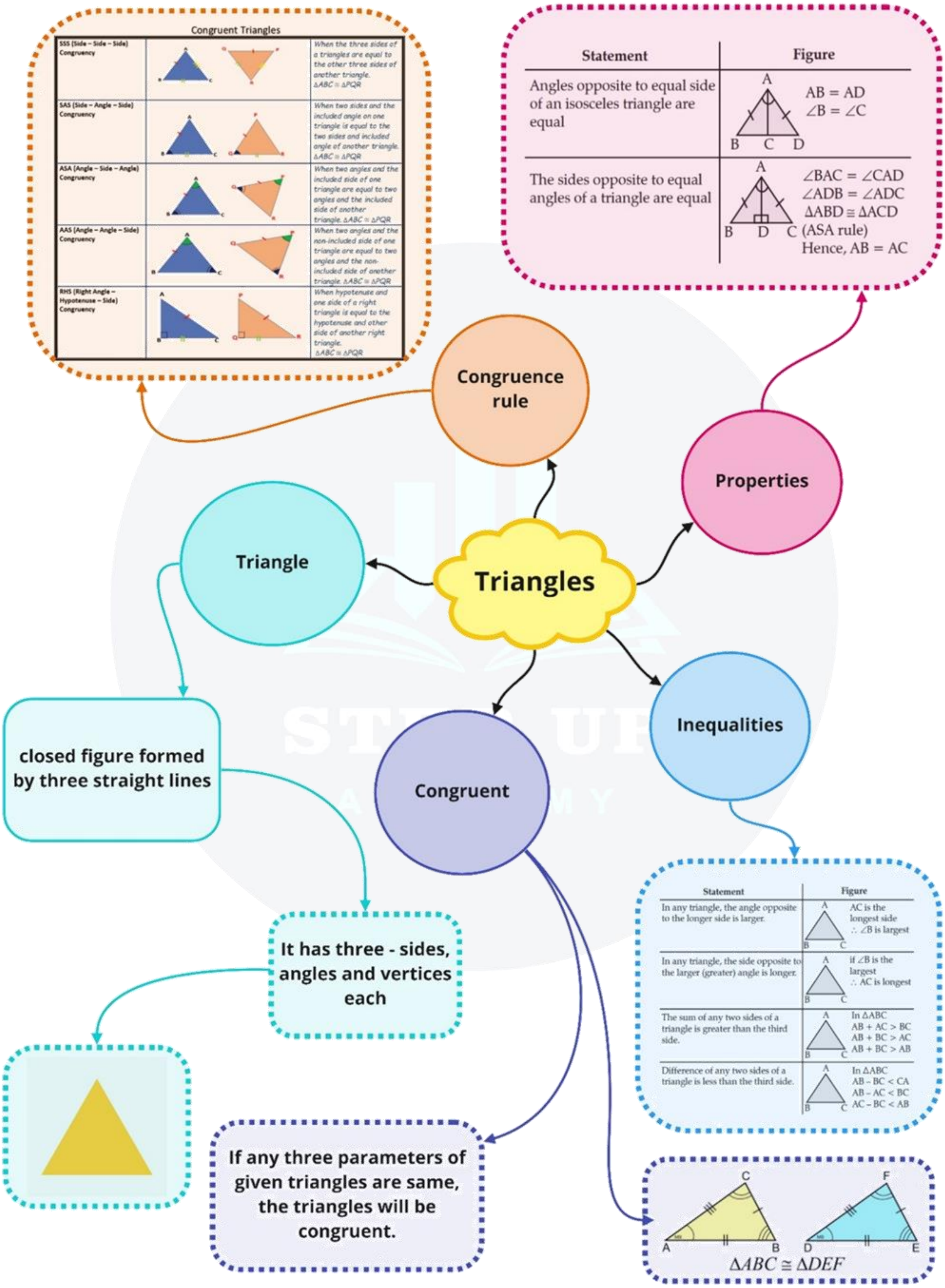
To Prove: |BC|< |AB| + |AC|

Construction: Consider a ΔABC. Extend the side AC to a point D such that AD = AB as shown in the fig. below.



Proof of triangle inequality theorem

Class : 9th mathematics
Chapter- 7: Triangles



Important Questions

Multiple Choice Questions

1. ΔABC = ΔPQR, then which of the following is true?
   (a) CB = QP
   (b) CA = RP
   (c) AC = RQ
   (d) AB = RP
2. In ΔABC and ΔDEF, AB = DE and ∠A = ∠D. Then two triangles will be congruent by SA axiom if:
   (a) BC = EF
   (b) AC = EF
   (c) AC = DE
   (d) BC = DE
3. In a right triangle, the longest side is:
   (a) Perpendicular
   (b) Hypotenuse
   (c) Base
   (d) None of the above
4. In ΔABC, if ∠A = 45° and ∠B = 70°, then the shortest and the longest sides of the triangle are respectively,
   (a) BC, AB
   (a) AB, AC
   (c) AB, BC
   (d) BC, AC
5. If the altitudes from vertices of a triangle to the opposite sides are equal, then the triangles is
   (a) Scalene
   (b) Isosceles
   (c) Equilateral
   (d) Right-angled
6. D is a Point on the Side BC of a ΔABC such that AD bisects ∠BAC then:
   (a) BD = CD
   (b) CD > CA
   (c) BD > BA
   (d) BA > BD
7. If ΔABC ≅ ΔPQR then which of the following is true:
   (a) CA = RP
   (b) AB = RP
   (c) AC = RQ
   (d) CB = QP
8. If two triangles ABC and PQR are congruent under the correspondence A ↔ P, B ↔ Q, and C ↔ R, then symbolically, it is expressed as
   (a) ΔABC ≅ ΔPQR
   (b) ΔABC = ΔPQR
   (c) ΔABC and ΔPQR are scalene triangles
   (d) ΔABC and ΔPQR are isosceles triangles
9. If the bisector of the angle A of an ΔABC is perpendicular to the base BC of the triangle then the triangle ABC is:
   (a) Obtuse Angled
   (b) Isosceles
   (c) Scalene
   (d) Equilateral
10. If AB = QR, BC=RP and CA = QP, then which of the following holds?
   (a) ΔBCA ≅ ΔPQR
   (b) ΔABC ≅ ΔPQR
   (c) ΔCBA ≅ ΔPQR
   (d) ΔCAB ≅ ΔPQR

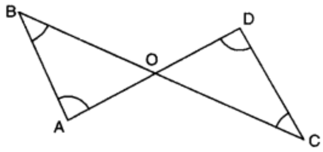
Very Short Questions:

1. Find the measure of each exterior angle of an equilateral triangle.
2. If in ΔABC, ∠A = ∠B + ∠C, then write the shape of the given triangle.
3. In ΔPQR, PQ = QR and ∠R = 50°, then find the measure of ∠Q.
4. If ΔSKY ≅ ΔMON by SSS congruence rule, then write three equalities of corresponding angles.
5. Is ΔABC possible, if AB = 6 cm, BC = 4 cm and AC = 1.5cm?
6. In ΔMNO, if ∠N = 90°, then write the longest side.
7. In ΔABC, if AB = AC and ∠B = 70°, find ∠A.
8. In ΔABC, if AD is a median, then show that AB + AC > 2AD.

Short Questions:

1. In the given figure, AD = BC and BD = AC, prove that ∠DAB = ∠CBA.
2. In the given figure, ΔABD and ΔBCD are isosceles triangles on the same base BD. Prove that ∠ABC = ∠ADC.
3. In the given figure, if ∠1 = ∠2 and ∠3 = ∠4, then prove that BC = CD.

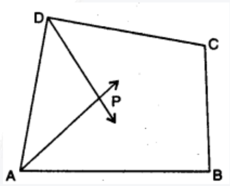
4. In the given figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.



5. In the given figure, $AC > AB$ and D is a point on AC such that $AB = AD$. Show that $BC > CD$.
6. In a triangle ABC, D is the mid-point of side AC such that $BD = \frac{1}{2}AC$ Show that $\angle ABC$ is a right angle.

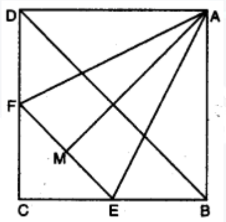
Long Questions:

1. In the given figure, AP and DP are bisectors of two adjacent angles A and D of quadrilateral ABCD. Prove that $2\angle APD = \angle B + 2C$



2. In figure, ABCD is a square and EF is parallel to diagonal BD and EM = FM. Prove that

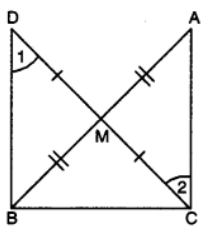
   (i) DF = BE (i) AM bisects $\angle BAD$.



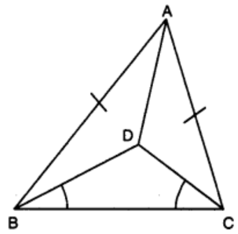
3. In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see fig.). Show that :

   (i) $\Delta AMC \cong \Delta BMD$ (ii) $\angle DBC = 90°$

   (ii) $\Delta DBC \cong \Delta ACB$ (iv) $CM = \frac{1}{2}AB$



4. In figure, ABC is an isosceles triangle with AB = AC. D is a point in the interior of ΔABC such that $\angle BCD = \angle CBD$. Prove that AD bisects $\angle BAC$ of ΔABC.



5. Prove that two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: If we draw two triangles with angles 30°, 70° and 80° and the length of the sides of one triangle be different than that of the corresponding sides of the other triangle then two triangles are not congruent.

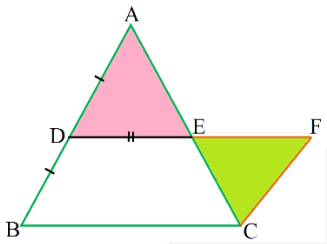
   Reason: If two triangles are constructed which have all corresponding angles equal but have unequal corresponding sides, then two triangles cannot be congruent to each other.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

Assertion: If the bisector of the vertical angle of a triangle bisects the base of the triangle, then the triangle is equilateral.

Reason: If three sides of one triangle are equal to three of the other triangle, then the two triangles are congruent.

Case Study Questions:

1. Read the Source/ Text given below and answer these questions:



Hareesh and Deep were trying to prove a theorem. For this they did the following:

i. Drew a triangle ABC.

ii. D and E are found as the mid points of AB and AC.

iii. DE was joined and DE was extended to F so DE = EF.

iv. FC was joined.

Answer the following questions:

i. ΔADE and ΔEFC are congruent by which criteria?
   a. SSS
   b. RHS
   c. SAS
   d. ASA

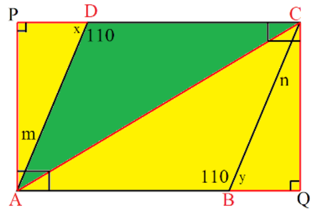
ii. ∠EFC is equal to which angle?
   a. ∠DAE
   b. ∠ADE
   c. ∠AED
   d. ∠B

iii. ∠ECF is equal to which angle?
   a. ∠DAE
   b. ∠ADE
   c. ∠AED
   d. ∠B

iv. CF is equal to which of the following?
   a. BD
   b. CE
   c. AE
   d. EF

v. CF is parallel to which of the following?
   a. AE
   b. CE
   c. BD
   d. EF

2. Read the Source/ Text given below and answer these questions:



In the middle of the city, there was a park ABCD in the form of a parallelogram form so that AB = CD, AB || CD and AD = BC, AD || BC Municipality converted this park into a rectangular form by adding land in the form of ΔAPD and ΔBCQ. Both the triangular shape of land were covered by planting flower plants.

Answer the following questions:

i. What is the value of ∠x?
   a. 110°
   b. 70°
   c. 90°
   d. 100°

ii. ΔAPD and ΔBCQ are congruent by which criteria?
   a. SSS
   b. SAS
   c. ASA
   d. RHS

iii. PD is equal to which side?
   a. DC
   b. AB
   c. BC
   d. BQ

iv. ΔABC and ΔACD are congruent by which criteria?
   a. SSS
   b. SAS
   c. ASA
   d. RHS

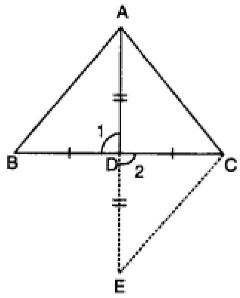
v. What is the value of ∠m?
   a. 110°
   b. 70°
   c. 90°
   d. 20°

Answer Key

Multiple Choice Questions

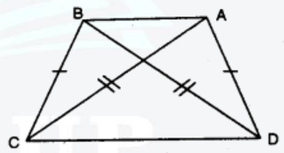
1. (b) CA = RP
2. (c) AC = DE
3. (b) Hypotenuse
4. (d) BC, AC
5. (b) Isosceles
6. (d) BA > BD
7. (a) CA = RP
8. (a) ΔABC ≅ ΔPQR
9. (b) Isosceles
10. (d) ΔCAB ≅ ΔPQR

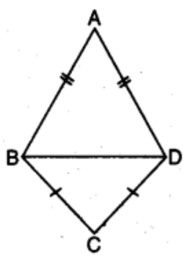
Very Short Answer:

1. We know that each interior angle of an equilateral triangle is 60°.
   ∴ Each exterior angle = 180° – 60° = 120°
2. Here, ∠A = ∠B + ∠C
   And in ΔABC, by angle sum property, we have
   ∠A + ∠B + C = 180°
   ⇒ ∠A + ∠A = 180°
   ⇒ 2∠A = 180°
   ⇒ ∠A = 90°
   Hence, the given triangle is a right triangle.
3. Here, in ΔPQR, PQ = QR
   ⇒ ∠R = ∠P = 50° (given)
   Now, ∠P + ∠Q + ∠R = 180°
   50° + ∠Q + 50° = 180°
   ⇒ ∠Q = 180° – 50° – 50°
   = 80°
4. Since ΔSKY ≅ ΔMON by SSS congruence rule, then three equalities of corresponding angles are ∠S = ∠M, ∠K = ∠O and ∠Y = ∠N
5. Since 4 + 1.5 = 5.5 ≠ 6
   Thus, triangle is not possible.
6. We know that, side opposite to the largest angle is longest.
   ∴ Longest side = MO.
7. Here, in ΔABC AB = AC ∠C = ∠B (∠s opp. to equal sides of a Δ)
   Now, ∠A + ∠B + ∠C = 180°
   ⇒ ∠A + 70° + 70° = 180°
   [∵ ∠B = 70°]
   ⇒ ∠A = 180° – 70° – 70° = 40°
8. 

   Produce AD to E, such that AD = DE.
   In ΔADB and ΔEDC, we have
   BD = CD, AD = DE and ∠1 = ∠2
   ΔADB ≅ ΔEDC
   AB = CE
   Now, in ΔAEC, we have
   AC + CE > AE
   AC + AB > AD + DE
   AB + AC > 2AD [∵ AD = DE]

Short Answer:

1. **Solution:**
   

   In ΔDAB and ΔCBA, we have
   AD = BC [given]
   BD = AC [given]
   AB = AB [common]
   ∴ ΔDAB ≅ ΔCBA [by SSS congruence axiom]
   Thus, ∠DAB =∠CBA [c.p.c.t.]
2. **Solution:**
   

   In ΔABD, we have
   AB = AD (given)
   ∠ABD = ∠ADB [angles opposite to equal sides are equal] ...(i)
   In ΔBCD, we have

CB = CD

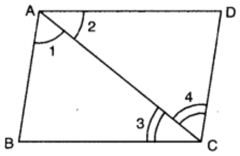
⇒ ∠CBD = ∠CDB [angles opposite to equal sides are equal] ... (ii)

Adding (i) and (ii), we have

∠ABD + ∠CBD = ∠ADB + ∠CDB

⇒ ∠ABC = ∠ADC

3. **Solution:**



In ΔABC and ACDA, we have

∠1 = ∠2 (given)

AC = AC [common]

∠3 = ∠4 [given]

So, by using ASA congruence axiom

ΔABC ≅ ΔCDA

Since corresponding parts of congruent triangles are equal

∴ BC = CD

4. **Solution:**

In ΔABC and ACDA, we have

∠1 = ∠2 (given)

AC = AC [common]

∠3 = ∠4 [given]

So, by using ASA congruence axiom

ΔABC ≅ ΔCDA

Since corresponding parts of congruent triangles are equal

∴ BC = CD

5. **Solution:**

Here, in ΔABD, AB = AD

∠ABD = ∠ADB

[∠s opp. to equal sides of a Δ]

In ΔBAD

ext. ∠BDC = ∠BAD + ∠ABD

⇒ ∠BDC > ∠ABD .... (ii)

Also, in ΔBDC.

ext. ∠ADB > ∠CBD ...(iii)

From (ii) and (iii), we have

∠BDC > CD [∵ sides opp. to greater angle is larger]

6. **Solution:**

Here, in ΔABC, D is the mid-point of AC.

⇒ AD = CD = $\frac{1}{2}$ AC ....(i)

Also, BD = $\frac{1}{2}$ AC .... (ii) [Given]

From (i) and (ii), we obtain

AD = BD and CD = BD

⇒ ∠2 = ∠4 and ∠1 = ∠3 .....(iii)

In ΔABC, we have

∠ABC + ∠ACB + ∠CAB = 180°

⇒ ∠1 + ∠2 + ∠3 + ∠4 = 180°

⇒ ∠1 + ∠2 + ∠1 + ∠2 = 180° [using (iii)]

⇒ 2(∠1 + ∠2) = 180°

⇒ ∠1 + ∠2 = 90°

Hence, ∠ABC = 90°

Long Answer:

1. **Solution:**

Here, AP and DP are angle bisectors of ∠A and ∠D

∴ ∠DAP = $\frac{1}{2}$ ∠DAB and ∠ADP = $\frac{1}{2}$ ∠ADC ......(i)

In ΔAPD, ∠APD + ∠DAP + ∠ADP = 180°

⇒ ∠APD + $\frac{1}{2}$ ∠DAB + $\frac{1}{2}$ ∠ADC = 180°

⇒ ∠APD = 180° – $\frac{1}{2}$ (∠DAB + ∠ADC)

⇒ 2∠APD = 360° – (∠DAB + ∠ADC) ......(ii)

Also, ∠A + ∠B + C + ∠D = 360°

∠B + 2C = 360° – (∠A + ∠D)

∠B + C = 360° – (∠DAB + ∠ADC) ......(iii)

From (ii) and (iii), we obtain

2∠APD = ∠B + ∠C

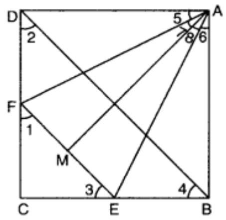
2. **Solution:**

(i) EF || BD = ∠1 = ∠2 and ∠3 = ∠4 [corresponding ∠s]

Also, ∠2 = ∠4 ⇒ ∠1 = ∠3

⇒ CE = CF (sides opp. to equals ∠s of a Δ]

∴ DF = BE [∵ BC – CE = CD – CF)



(ii) In ΔADF and ΔABE, we have

AD = AB [sides of a square]

DF = BE [proved above]

∠D = ∠B = 90°

⇒ ΔADF ≅ ΔABE [by SAS congruence axiom]

$\Rightarrow$ AF = AE and $\angle 5 = \angle 6$ ... (i) [c.p.c.t.]

In ΔAMF and ΔAME

AF = AE [proved above]

AM = AM [common]

FM = EM (given)

$\therefore$ $\Delta AMF \cong \Delta AME$ [by SSS congruence axiom]

$\therefore$ $\angle 7 = \angle 8$ ...(ii) [c.p.c.t.]

Adding (i) and (ii), we have

$\angle 5 + \angle 7 = \angle 6 + \angle 8$

$\angle DAM = \angle BAM$

$\therefore$ AM bisects $\angle BAD$.

3. **Solution:**

Given: ΔACB in which 4C = 90° and M is the mid-point of AB.

To Prove:

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC = 90°$

(iii) $\Delta DBC \cong \Delta ACB$

(iv) CM =1/2 AB

Proof: Consider ΔAMC and ΔBMD,

we have AM = BM [given]

CM = DM [by construction]

$\angle AMC = \angle BMD$ [vertically opposite angles]

$\therefore$ $\Delta AMC \cong \Delta BMD$ [by SAS congruence axiom]

$\Rightarrow$ AC = DB ...(i) [by c.p.c.t.]

and $\angle 1 = \angle 2$ [by c.p.c.t.]

But $\angle 1$ and $\angle 2$ are alternate angles.

$\Rightarrow$ BD || CA

Now, BD || CA and BC is transversal.

$\therefore$ $\angle ACB + \angle CBD = 180°$

$\Rightarrow$ 90° + CBD = 180°

$\Rightarrow$ $\angle CBD = 90°$

In ΔDBC and ΔACB,

we have CB = BC [common]

DB = AC [using (i)]

$\angle CBD = \angle BCA$

$\therefore$ $\Delta DBC \cong \Delta ACB$

$\Rightarrow$ DC = AB

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow \frac{1}{2}AB = CM$ Or $CM = \frac{1}{2}AB$ ($\because CM = \frac{1}{2}DC$)

4. **Solution:**

In ΔBDC, we have $\angle DBC = \angle DCB$ (given).

$\Rightarrow$ CD = BD (sides opp. to equal $\angle$s of ΔDBC)

Now, in ΔABD and ΔACD,

we have AB = AC [given]

BD = CD [proved above]

AD = AD [common]

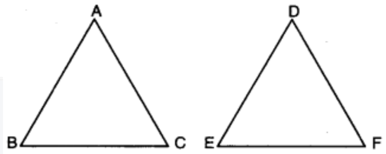
$\therefore$ By using SSS congruence axiom, we obtain

$\Delta ABD \cong \Delta ACD$

$\Rightarrow$ $\angle BAD = \angle CAD$ [c.p.ç.t.]

Hence, AD bisects $\angle BAC$ of ΔABC.

5. **Solution:**



Given: Two As ABC and DEF in which

$\angle B = \angle E$,

$\angle C = \angle F$ and BC = EF

To Prove: $\Delta ABC = \Delta DEF$

Proof: We have three possibilities

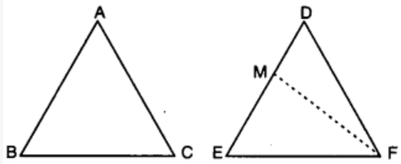
Case I. If AB = DE,

we have AB = DE,

$\angle B = \angle E$ and BC = EF.

So, by SAS congruence axiom, we have

$\Delta ABC \cong \Delta DEF$



Case II. If AB < ED, then take a point Mon ED such that EM = AB.

Join MF.

Now, in ΔABC and ΔMEF,

we have

AB = ME, $\angle B = \angle E$ and BC = EF.

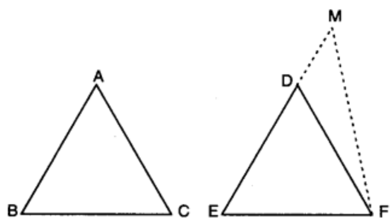
So, by SAS congruence axiom,

we have $\Delta ABC \cong \Delta MEF$

$\Rightarrow$ $\angle ACB = \angle MFE$

But $\angle ACB = \angle DFE$

$\therefore$ $\angle MFE = \angle DFE$



Which is possible only when FM coincides with B FD i.e., M coincides with D.

Thus, AB = DE

∴ In ΔABC and ΔDEF, we have

AB = DE,

∠B = ∠E and BC = EF

So, by SAS congruence axiom, we have

ΔABC ≅ ΔDEF

Case III. When AB > ED

Take a point M on ED produced

such that EM = AB.

Join MF

Proceeding as in Case II, we can prove that

ΔABC = ΔDEF

Hence, in all cases, we have

ΔABC = ΔDEF.

Assertion and Reason Answers-

1. (b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Case Study Answer:

1.

(i)	(c)	SAS
(ii)	(b)	∠ADE∠ADE
(iii)	(a)	∠DAE∠DAE
(iv)	(a)	BD
(v)	(c)	BD

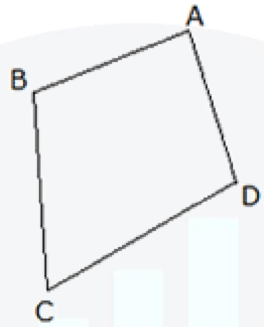
2.

(i)	(b)	70°
(ii)	(c)	ASA
(iii)	(d)	BQ
(iv)	(a)	SSS
(v)	(d)	20°

Quadrilaterals 8

Quadrilateral

A **quadrilateral** is a closed figure obtained by joining four points (with no three points collinear) in an order.



Here, ABCD is a quadrilateral.

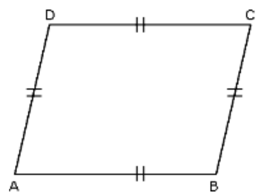
Parts of a quadrilateral

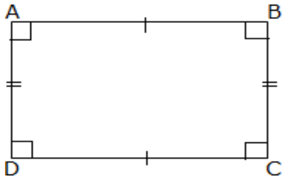
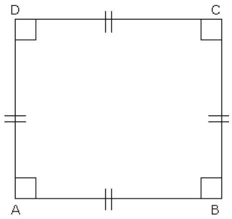
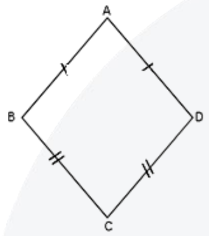
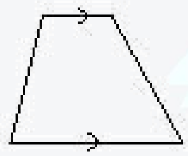
- A quadrilateral has four sides, four angles and four vertices.
- Two sides of a quadrilateral having no common end point are called its **opposite sides**.
- Two sides of a quadrilateral having a common end point are called its **adjacent sides**.
- Two angles of a quadrilateral having common arm are called its **adjacent angles**.
- Two angles of a quadrilateral not having a common arm are called its **opposite angles**.
- A **diagonal** is a line segment obtained on joining the opposite vertices.

Angle sum property of a quadrilateral

Sum of all the angles of a quadrilateral is 360o. This is known as the **angle sum property of a quadrilateral**.

Types of quadrilaterals and their properties:

Name of a quadrilateral	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	i. Opposite sides are equal. ii. Opposite angles are equal. iii. Diagonals bisect one another.
Rhombus: A parallelogram with sides of equal length. 	i. All properties of a parallelogram. ii. Diagonals are perpendicular to each other.

Rectangle: A parallelogram with all angles right angle.  	i. All the properties of a parallelogram. ii. Each of the angles is a right angle. iii. Diagonals are equal.
Square: A rectangle with sides of equal length.  	All the properties of a parallelogram, a rhombus and a rectangle.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides.  	i. The diagonals are perpendicular to one another. ii. One of the diagonals bisects the other. iii. If ABCD is a kite, then $\angle B = \angle D$ but $\angle A \neq \angle C$
Trapezium: A quadrilateral with one pair of opposite sides parallel is called trapezium. 	One pair of opposite sides parallel.

Important facts about quadrilaterals

- If the non-parallel sides of trapezium are equal, it is known as isosceles trapezium.
- Square, rectangle and rhombus are all parallelograms.
- Kite and trapezium are not parallelograms.
- A square is a rectangle.
- A square is a rhombus.
- A parallelogram is a trapezium.

A quadrilateral is a parallelogram if:

i. each pair of opposite sides of a quadrilateral is equal, or
ii. each pair of opposite angles is equal, or
iii. the diagonals of a quadrilateral bisect other, or
iv. each pair of opposite sides is equal and parallel.

Mid-Point Theorem

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Converse of mid-point theorem

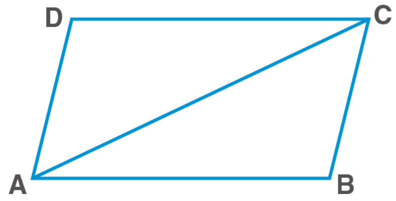
The line drawn through the mid-point of one side of a triangle, parallel to another side, bisects the third side.

Formation of a new quadrilateral using the given data

- If the diagonals of a parallelogram are equal, then it is a rectangle.
- If the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.
- If the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

Parallelogram: Opposite sides of a parallelogram are equal



In ΔABC and ΔCDA

AC = AC [Common / transversal]

∠BCA = ∠DAC [alternate angles]

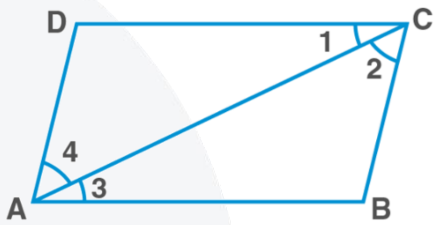
∠BAC = ∠DCA [alternate angles]

ΔABC ≅ ΔCDA [ASA rule]

Hence,

AB = DC and AD= BC [C.P.C.T.C]

Opposite angles in a parallelogram are equal



In parallelogram ABCD

AB || CD; and AC is the transversal

Hence, ∠1 = ∠3 .... (1) (alternate interior angles)

BC || DA; and AC is the transversal

Hence, ∠2 = ∠4 .... (2) (alternate interior angles)

Adding (1) and (2)

∠1 + ∠2 = ∠3 + ∠4

∠BAD = ∠BCD

Similarly,

∠ADC = ∠ABC

Properties of diagonal of a parallelogram

Diagonals of a parallelogram bisect each other.

In ΔAOB and ΔCOD,

∠3 = ∠5 [alternate interior angles]

∠1 = ∠2 [vertically opposite angles]

AB = CD [opp. Sides of parallelogram]

ΔAOB ≅ ΔCOD [AAS rule]

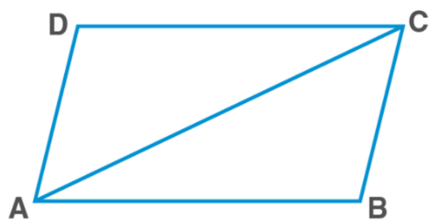
OB = OD and OA = OC [C.P.C.T]

Hence, proved

Conversely,

If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Diagonal of a parallelogram divides it into two congruent triangles.



In ΔABC and ΔCDA,

AB = CD [Opposite sides of parallelogram]

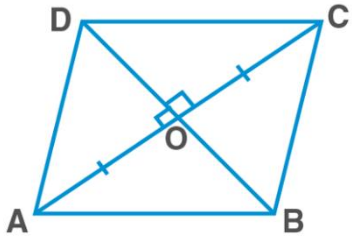
BC = AD [Opposite sides of parallelogram]

AC = AC [Common side]

ΔABC ≅ ΔCDA [by SSS rule]

Hence, proved.

Diagonals of a rhombus bisect each other at right angles



Diagonals of a rhombus bisect each – other at right angles

In ΔAOD and ΔCOD,

OA = OC [Diagonals of parallelogram bisect each other]

OD = OD [Common side]

AD = CD [Adjacent sides of a rhombus]

ΔAOD ≅ ΔCOD [SSS rule]

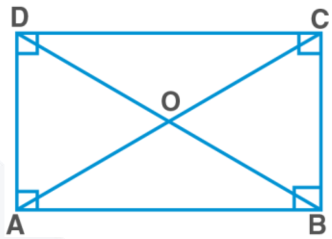
∠AOD = ∠DOC [C.P.C.T]

∠AOD + ∠DOC = 180 [∵ AOC is a straight line]

Hence, ∠AOD = ∠DOC = 90

Hence proved.

Diagonals of a rectangle bisect each other and are equal



Rectangle ABCD

In ΔABC and ΔBAD,

AB = BA [Common side]

BC = AD [Opposite sides of a rectangle]

∠ABC = ∠BAD [Each = 900 ∵ ABCD is a Rectangle]

ΔABC ≅ ΔBAD [SAS rule]

∴AC = BD [C.P.C.T]

Consider ΔOAD and ΔOCB,

AD = CB [Opposite sides of a rectangle]

∠OAD = ∠OCB [∵ AD||BC and transversal AC intersects them]

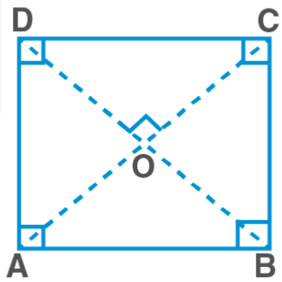
∠ODA = ∠OBC [∵ AD||BC and transversal BD intersects them]

ΔOAD ≅ ΔOCB [ASA rule]

∴OA = OC [C.P.C.T]

Similarly, we can prove OB=OD

Diagonals of a square bisect each other at right angles and are equal



In ΔABC and ΔBAD,

AB = BA [Common side]

BC = AD [Opposite sides of a Square]

∠ABC = ∠BAD [Each = 900 ∵ ABCD is a Square]

ΔABC ≅ ΔBAD [SAS rule]

∴ AC = BD [C.P.C.T]

Consider ΔOAD and ΔOCB,

AD = CB [Opposite sides of a Square]

∠OAD = ∠OCB [∵ AD||BC and transversal AC intersects them]

∠ODA = ∠OBC [∵ AD||BC and transversal BD intersects them]

ΔOAD ≅ ΔOCB [ASA rule]

∴ OA = OC [C.P.C.T]

Similarly, we can prove OB=OD

In ΔOBA and ΔODA,

OB = OD [proved above]

BA = DA [Sides of a Square]

OA = OA [Common side]

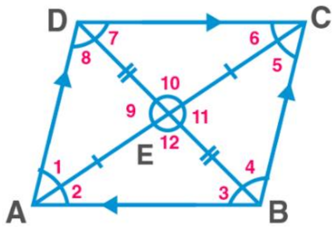
ΔOBA ≅ ΔODA, [SSS rule]

∴ ∠AOB = ∠AOD [C.P.C.T]

But ∠AOB + ∠AOD = 1800 [Linear pair]

∴ ∠AOB = ∠AOD = 900

Important results related to parallelograms



Opposite sides of a parallelogram are parallel and equal.

AB||CD, AD||BC, AB = CD, AD = BC

Opposite angles of a parallelogram are equal adjacent angels are supplementary.

∠A = ∠C, ∠B = ∠D,

∠A + ∠B = 1800, ∠B + ∠C = 1800, ∠C + ∠D = 1800, ∠D + ∠A = 1800

A diagonal of parallelogram divides it into two congruent triangles.

ΔABC ≅ ΔCDA [With respect to AC as diagonal]

ΔADB ≅ ΔCBD [With respect to BD as diagonal]

The diagonals of a parallelogram bisect each other.

AE = CE, BE = DE

∠1 = ∠5 (alternate interior angles)

∠2 = ∠6 (alternate interior angles)

∠3 = ∠7 (alternate interior angles)

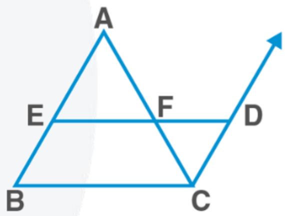
∠4 = ∠8 (alternate interior angles)

∠9 = ∠11 (vertically opp. angles)

∠10=∠12 (vertically opp. angles)

The Mid-Point Theorem

The line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half of the third side



In ΔABC, E – the midpoint of AB; F – the midpoint of AC

Construction: Produce EF to D such that EF = DF.

In ΔAEF and ΔCDF,

AF = CF [F is the midpoint of AC]

∠AFE = ∠CFD [V.O.A]

EF = DF [Construction]

∴ ΔAEF ≅ ΔCDF [SAS rule]

Hence,

∠EAF = ∠DCF .... (1)

DC = EA = EB [E is the midpoint of AB]

DC || EA || AB [Since, (1), alternate interior angles]

DC || EB

So EBCD is a parallelogram

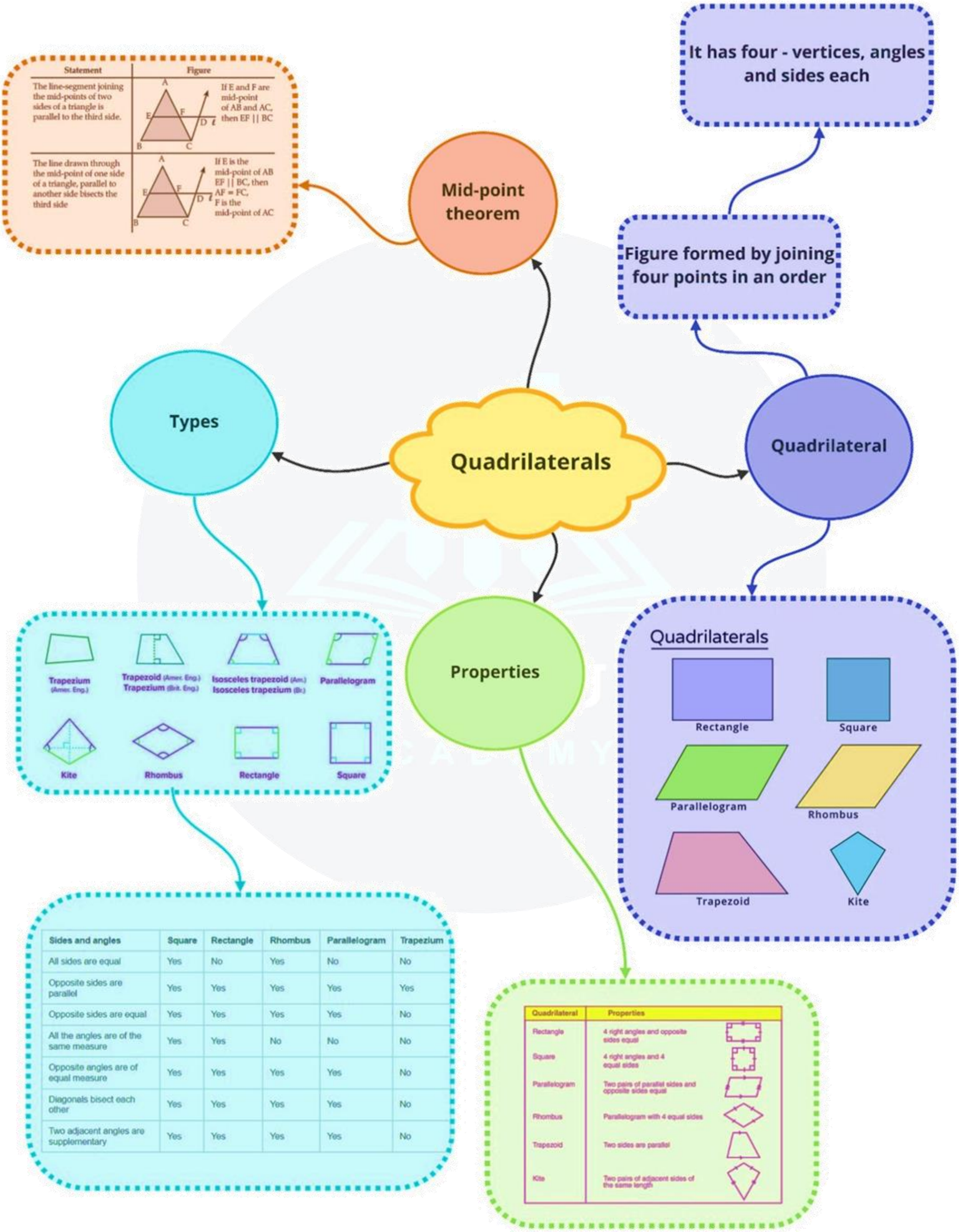
Therefore, BC = ED and BC || ED

Since ED = EF + FD = 2EF = BC [∵ EF=FD]

We have, EF = 12 BC and EF||BC

Hence proved.

Class : 9th mathematics
Chapter- 8: Quadrilaterals



Sides and angles	Square	Rectangle	Rhombus	Parallelogram	Trapezium
All sides are equal	Yes	No	Yes	No	No
Opposite sides are parallel	Yes	Yes	Yes	Yes	Yes
Opposite sides are equal	Yes	Yes	Yes	Yes	No
All the angles are of the same measure	Yes	Yes	No	No	No
Opposite angles are of equal measure	Yes	Yes	Yes	Yes	No
Diagonals bisect each other	Yes	Yes	Yes	Yes	No
Two adjacent angles are supplementary	Yes	Yes	Yes	Yes	No

Quadrilateral	Properties
Rectangle	4 right angles and opposite sides equal
Square	4 right angles and 4 equal sides
Parallelogram	Two pairs of parallel sides and opposite sides equal
Rhombus	Parallelogram with 4 equal sides
Trapezoid	Two sides are parallel
Kite	Two pairs of adjacent sides of the same length

Important Questions

Multiple Choice Questions

1. A diagonal of a Rectangle is inclines to one side of the rectangle at an angle of 25°. The Acute Angle between the diagonals is:
   (a) 115°
   (b) 50°
   (c) 40°
   (d) 25°
2. The diagonals of a rectangle PQRS intersects at O. If ∠QOR = 44°, ∠OPS =?
   (a) 82°
   (b) 52°
   (c) 68°
   (d) 75°
3. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is
   (a) Rhombus
   (b) Parallelogram
   (c) Trapezium
   (d) Kite
4. All the angles of a convex quadrilateral are congruent. However, not all its sides are congruent. What type of quadrilateral is it?
   (a) Parallelogram
   (b) Square
   (c) Rectangle
   (d) Trapezium
5. In a Quadrilateral ABCD, AB = BC and CD = DA, then the quadrilateral is a
   (a) Triangle
   (b) Kite
   (c) Rhombus
   (d) Rectangle
6. The angles of a quadrilateral are (5x)°, (3x + 10)°, (6x – 20)° and (x + 25)°. Now, the measure of each angle of the quadrilateral will be
   (a) 115°, 79°, 118°, 48°
   (b) 100° 79°, 118°, 63°
   (c) 110°, 84°, 106°, 60°
   (d) 75°, 89°, 128°, 68°
7. The diagonals of rhombus are 12 cm and 16 cm. The length of the side of rhombus is:
   (a) 12cm
   (b) 16cm
   (c) 8cm
   (d) 10cm
8. In quadrilateral PQRS, if ∠P = 60° and ∠Q:∠R:∠S= 2 : 3 : 7, then ∠S =
   (a) 175°
   (b) 210°
   (c) 150°
   (d) 135°
9. In parallelogram ABCD, if ∠A = 2x + 15°, ∠B = 3x – 25°, then value of x is:
   (a) 91°
   (b) 89°
   (c) 34°
   (d) 38°
10. If ABCD is a trapezium in which AB || CD and AD = BC, then:
    (a) ∠A = ∠B
    (b) ∠A > ∠B
    (c) ∠A < ∠B
    (d) None of the above

Very Short Questions:

1. If one angle of a parallelogram is twice of its adjacent angle, find the angles of the parallelogram.
2. If the diagonals of a quadrilateral bisect each other at right angles, then name the quadrilateral.
3. Three angles of a quadrilateral are equal, and the fourth angle is equal to 144°. Find each of the equal angles of the quadrilateral.
4. If ABCD is a parallelogram, then what is the measure of ∠A – ∠C?
5. PQRS is a parallelogram, in which PQ = 12 cm and its perimeter is 40 cm. Find the length of each side of the parallelogram.
6. Two consecutive angles of a parallelogram are (x + 60)° and (2x + 30)°. What special name can you give to this parallelogram?
7. ONKA is a square with ∠KON = 45°. Determine ∠KOA.

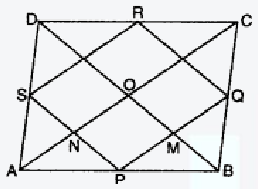
8. In quadrilateral PQRS, if ∠P = 60° and ∠Q : ∠R : ∠S = 2 : 3 : 7, then find the measure of ∠S.

Short Questions:

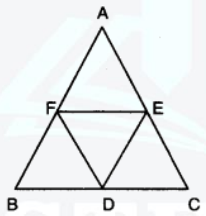
1. ABCD is a parallelogram in which ∠ADC = 75° and side AB is produced to point E as shown in the figure. Find x + y.
2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.
3. In the figure, ABCD is a rhombus, whose diagonals meet at O. Find the values of x and y.
4. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see fig.). Show that:

   (i) ΔAPB = ΔCQD

   (ii) AP = CQ
5. The diagonals of a quadrilateral ABCD are perpendicular to each other. Show that the quadrilateral formed by joining the mid-points of its sides is a rectangle.

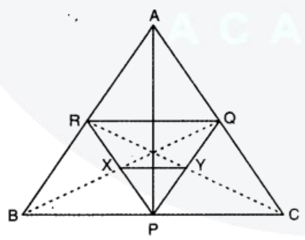


6. In the fig., D, E and F are, respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC. Prove that DEF is also an equilateral triangle.

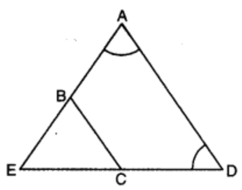


Long Questions:

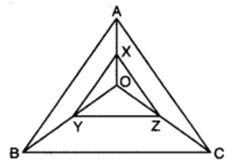
1. In the figure, P, Q and R are the mid-points of the sides BC, AC and AB of ΔABC. If BQ and PR intersect at X and CR and PQ intersect at Y, then show that XY = $\frac{1}{4}$ BC



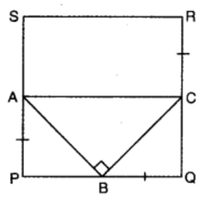
2. In the given figure, AE = DE and BC || AD. Prove that the points A, B, C and D are concyclic. Also, prove that the diagonals of the quadrilateral ABCD are equal.



3. In ΔABC, AB = 8cm, BC = 9 cm and AC = 10cm. X, Y and Z are mid-points of AO, BO and CO respectively as shown in the figure. Find the lengths of the sides of ΔXYZ.



4. PQRS is a square and ∠ABC = 90° as shown in the figure. If AP=BQ=CR, then prove that ∠BAC = 45°



5. ABCD is a parallelogram. If the bisectors DP and CP of angles D and C meet at P on side AB, then show that P is the mid-point of side AB.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

   c) Assertion is correct statement but reason is wrong statement.

   d) Assertion is wrong statement but reason is correct statement.

   Assertion: ABCD is a square. AC and BD intersect at O. The measure of +AOB = 90°.

   Reason: Diagonals of a square bisect each other at right angles.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

   c) Assertion is correct statement but reason is wrong statement.

   d) Assertion is wrong statement but reason is correct statement.

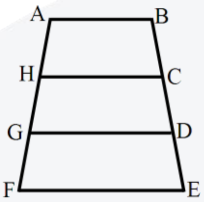
Assertion: The consecutive sides of a quadrilateral have one common point.

Reason: The opposite sides of a quadrilateral have two common point.

Case Study Questions:

1. Read the Source/ Text given below and answer these questions:

Sohan wants to show gratitude towards his teacher by giving her a card made by him. He has three pieces of trapezium pasted one above the other as shown in fig. These pieces are arranged in a way that AB || HC || GD || FE. Also BC=CD=DE and AH=HG=GF=6cm. He wants to decorate the card by putting up a colored tape on the nonparallel sides of the trapezium.



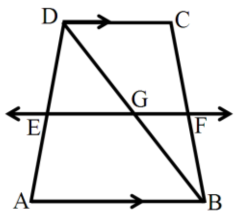
i. Find the total length of colored tape required if DE = 4cm.
   a. 20cm
   b. 30cm
   c. 40cm
   d. 50cm

ii. ABHC is a trapezium in which AB || HC and ∠A=∠B=45°. Find angles C and H of the trapezium.
   a. 135, 130
   b. 130, 135
   c. 135, 135
   d. 130, 130

iii. What is the difference between trapezium and parallelogram?
   a. Trapezium has 2 sides, and parallelogram has 4 sides.
   b. Trapezium has 4 sides, and parallelogram has 2 sides.
   c. Trapezium has 1 pair of parallel sides, and parallelogram has 2 pairs of parallel sides.
   d. Trapezium has 2 pairs of parallel sides, and parallelogram has 1 pair of parallel sides.

iv. Diagonals in isosceles trapezoid are _______.
   a. parallel.
   b. opposite.
   c. vertical.
   d. equal.

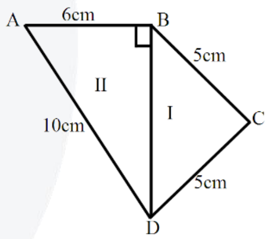
v. ABCD is a trapezium where AB || DC, BD is the diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F. Which of these is true?



   a. BF = FC
   b. EA = FB
   c. CF = DE
   d. None of these

2. Read the Source/ Text given below and answer any four questions:

Chocolate is in the form of a quadrilateral with sides 6cm and 10cm, 5cm and 5cm (as shown in the figure) is cut into two parts on one of its diagonal by a lady. Part-I is given to her maid and part II is equally divided among a driver and gardener.



i. Length of BD:
   a. 9cm
   b. 8cm
   c. 7cm
   d. 6cm

ii. Area of △ABC:
   a. $24cm^2$
   b. $12cm^2$
   c. $42cm^2$
   d. $21cm^2$

iii. The sum of all the angles of a quadrilateral is equal to:
   a. 180°
   b. 270°
   c. 360°
   d. 90°

iv. A diagonal of a parallelogram divides it into two congruent:
  a. Square.
  b. Parallelogram.
  c. Triangles.
  d. Rectangle.

v. Each angle of the rectangle is:
  a. More than 90°
  b. Less than 90°
  c. Equal to 90°
  d. Equal to 45°

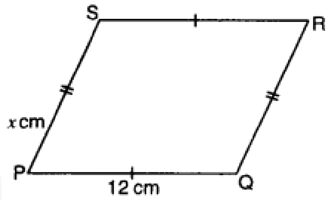
Answer Key

Multiple Choice Questions

1. (b) 50°
2. (c) 68°
3. (c) Trapezium
4. (c) Rectangle
5. (b) Kite
6. (a) 115°, 79°, 118°, 48°
7. (d) 10cm
8. (a) 175°
9. (d) 38°
10. (a) ∠A = ∠B

Very Short Answer:

1. Let the two adjacent angles be x and 2x.
   In a parallelogram, sum of the adjacent angles are 180°
   ∴ x + 2x = 180°
   ⇒ 3x = 180°
   ⇒ x = 60°
   Thus, the two adjacent angles are 120° and 60°. Hence, the angles of the parallelogram are 120°, 60°, 120° and 60°.
2. Rhombus.
3. Let each equal angle of given quadrilateral be x.
   We know that sum of all interior angles of a quadrilateral is 360°
   ∴ x + x + x + 144° = 360°
   3x = 360° – 144°
   3x = 216°
   x = 72°
   Hence, each equal angle of the quadrilateral is of 72° measures.
4. ∠A – ∠C = 0° (opposite angles of parallelogram are equal]
5. Here, PQ = SR = 12 cm
   Let PS = x and PS = QR
   ∴ x + 12 + x + 12 = Perimeter
   2x + 24 = 40
   2x = 16
   x = 8



   Hence, length of each side of the parallelogram is 12cm, 8 cm, 12cm and 8cm.
6. We know that consecutive interior angles of a parallelogram are supplementary.
   ∴ (x + 60° + (2x + 30)° = 180°
   ⇒ 3x° + 90° = 180°
   ⇒ 3x° = 90°
   ⇒ x° = 30°
   Thus, two consecutive angles are (30 + 60)°, 12 x 30 + 30)”. i.e., 90° and 90°.
   Hence, the special name of the given parallelogram is rectangle.
   7. Since ONKA is a square
   ∴ ∠AON = 90°
   We know that diagonal of a square bisects its ∠s
   ⇒ ∠AOK = ∠KON = 45°
   Hence, ∠KOA = 45°
   Now, ∠A + ∠B + ∠C = 180°
   ⇒ ∠A + 70° + 70° = 180°
   [∵ ∠B = 70°]
   ⇒ ∠A = 180° – 70° – 70° = 40°
7. Since ONKA is a square
   ∴ ∠AON = 90°
   We know that diagonal of a square bisects its ∠s
   ⇒ ∠AOK = ∠KON = 45°
   Hence, ∠KOA = 45°
   Now, ∠A + ∠B + ∠C = 180°
   ⇒ ∠A + 70° + 70° = 180°
   [∵ ∠B = 70°]
   ⇒ ∠A = 180° – 70° – 70° = 40°

8. Let ∠Q = 2x, ∠R = 3x and ∠S = 7x

Now, ∠P + ∠Q + ∠R + ∠S = 360°

⇒ 60° + 2x + 3x + 7x = 360°

⇒ 12x = 300°

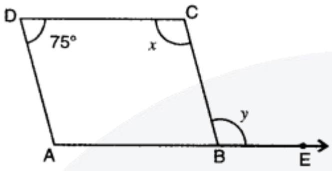
$x = \frac{300°}{12} = 25°$

∠S = 7x = 7 x 25° = 175°

Short Answer:

1. **Solution:**

Here, ∠C and ∠D are adjacent angles of the parallelogram.



∴ ∠C + ∠D = 180°

⇒ x + 75° = 180°

⇒ x = 105°

Also, y = x = 105°

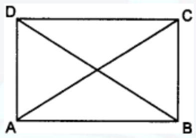
[alt. int. angles]

Thus, x + y = 105° + 105° = 210°

2. **Solution:**

Given: A parallelogram ABCD, in which AC = BD.

To Prove: ΔBCD is a rectangle.



Proof: In ΔABC and ΔBAD

AB = AB (common)

AC = BD (given)

BC = AD (opp. sides of a ||gm)

⇒ ΔABC ≅ ΔBAD

[by SSS congruence axiom]

⇒ ∠ABC = ∠BAD (c.p.c.t.)

Also, ∠ABC + ∠BAD = 180° (co-interior angles)

∠ABC + ∠ABC = 180° [∵ ∠ABC = ∠BAD]

2∠ABC = 180°

∠ABC = 1/2 x 180° = 90°

3. **Solution:**

Since diagonals of a rhombus bisect each other at right angle.

In ∴ ΔAOB, we have

∠OAB + ∠x + 90° = 180°

∠x = 180° – 90° – 35°

= 55°

Also,

∠DAO = ∠BAO = 35°

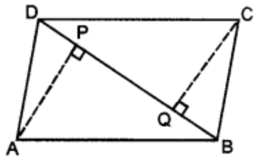
∠y + ∠DAO + ∠BAO + ∠x = 180°

⇒ ∠y + 35° + 35° + 55° = 180°

⇒ ∠y = 180° – 125o = 55°

Hence, the values of x and y are x = 55°, y = 55°

4. **Solution:**



Given: In ||gm ABCD, AP and CQ are perpendiculars from the vertices A and C on the diagonal BD.

To Prove: (i) ΔAPB ≅ ΔCQD

(ii) AP = CQ

Proof: (i) In ΔAPB and ΔCQD

AB = DC (opp. sides of a ||gm ABCD]

∠APB = ∠DQC (each = 90°)

∠ABP = ∠CDQ (alt. int. ∠s]

⇒ ΔAPB ≅ ΔCQD [by AAS congruence axiom]

(ii) ⇒ AP = CQ [c.p.c.t.]

5. **Solution:**

Given: A quadrilateral ABCD whose diagonals AC and BD are perpendicular to each other at O. P, Q, R and S are mid-points of side AB, BC, CD and DA respectively are joined are formed quadrilateral PQRS.

To Prove: PQRS is a rectangle.

Proof: In ΔABC, P and Q are mid-points of AB and BC respectively.

∴ PQ || AC and PQ = $\frac{1}{2}$ AC ... (i) (mid-point theorem]

Further, in SACD, R and S are mid-points of CD and DA respectively.

SR || AC and SR = $\frac{1}{2}$ AC ... (ii) (mid-point theorem]

From (i) and (ii), we have PQ || SR and PQ = SR

Thus, one pair of opposite sides of quadrilateral PQRS are parallel and equal.

∴ PQRS is a parallelogram.

Since PQ|| AC PM || NO

In ΔABD, P and S are mid-points of AB and AD respectively.

PS || BD (mid-point theorem]

⇒ PN || MO

∴ Opposite sides of quadrilateral PMON are parallel.

∴ PMON is a parallelogram.

∠ MPN = ∠ MON (opposite angles of ||gm are equal]

But ∠MON = 90° [given]

∴ ∠MPN = 90° ⇒ ∠QPS = 90°

Thus, PQRS is a parallelogram whose one angle is 90°

∴ PQRS is a rectangle.

6. **Solution:**

Since line segment joining the mid-points of two sides of a triangle is half of the third side.

Therefore, D and E are mid-points of BC and AC respectively.

$\Rightarrow DE = \frac{1}{2} AB$ ...(i)

E and F are the mid-points of AC and AB respectively.

$\therefore EF = \frac{1}{2} BC$ ... (ii)

F and D are the mid-points of AB and BC respectively.

$\therefore FD = \frac{1}{2} AC$ ... (iii)

Now, SABC is an equilateral triangle.

⇒ AB = BC = CA

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$

⇒ DE = EF = FD (using (i), (ii) and (iii)]

Hence, DEF is an equilateral triangle

Long Answer:

1. **Solution:**

Here, in ΔABC, R and Q are the mid-points of AB and AC respectively.

∴ By using mid-point theorem, we have

$RQ \parallel BC$ and $RQ = \frac{1}{2} BC$

∴ RQ = BP = PC [∵ P is the mid-point of BC]

∴ RQ || BP and RQ || PC

In quadrilateral BPQR

RQ || BP, RQ = BP (proved above]

∴ BPQR is a parallelogram. [∵ one pair of opp. sides is parallel as well as equal]

∴ X is the mid-point of PR. [∵ diagonals of a ||gm bisect each other]

Now, in quadrilateral PCQR

RQ || PC and RQ = PC [proved above)

∴ PCQR is a parallelogram [∵ one pair of opp. sides is parallel as well as equal]

∴ Y is the mid-point of PQ [∵ diagonals of a ||gm bisect each other]

In ΔPQR

∴ X and Y are mid-points of PR and PQ respectively.

$\therefore XY \parallel RQ$ and $XY = \frac{1}{2} RQ$ [by using mid-point theorem]

$XY = \frac{1}{2} RQ\,(\frac{1}{2} BC)$ [$\because RQ = \frac{1}{2} BC$]

$\Rightarrow XY = \frac{1}{4} BC$

2. **Solution:**

Since AE = DE

∠D = ∠A .... (i) [∵ ∠s opp. to equal sides of a Δ]

Again, BC || AD

∠EBC = ∠A .... (ii) (corresponding ∠s]

From (i) and (ii), we have

∠D = ∠EBC .... (iii)

But ∠EBC + ∠ABC = 180° (a linear pair]

∠D + ∠ABC = 180° (using (iii)]

Now, a pair of opposite angles of quadrilateral ABCD is supplementary

Thus, ABCD is a cyclic quadrilateral i.e., A, B, C and D'are concyclic. In ΔABD and ΔDCA

∠ABD = ∠ACD [∠s in the same segment for cyclic quad. ABCD]

∠BAD = ∠CDA [using (i)]

AD = AD (common]

So, by using AAS congruence axiom, we have

ΔABD ≅ ΔDCA

Hence, BD = CA [c.p.c.t.]

3. **Solution:**

Here, in ΔABC, AB = 8cm, BC = 9cm, AC = 10cm.

In ΔAOB, X and Y are the mid-points of AO and BO.

∴ By using mid-point theorem, we have

$XY = \frac{1}{2} AB = \frac{1}{2} \times 8cm = 4cm$

Similarly, in ΔτBOC, Y and Z are the mid-points of BO and CO.

∴ By using mid-point theorem, we have

$YZ = \frac{1}{2} BC = \frac{1}{2} \times 9cm = 4.5cm$

And, in ΔτCOA, Z and X are the mid-points of CO and AO.

$\therefore ZX = \frac{1}{2} AC = \frac{1}{2} \times 10cm = 5cm$

4. Solution:

Since PQRS is a square.

∴ PQ = QR ... (I) [∵ sides of a square are equal]

Also, BQ = CR ... (ii) [given]

Subtracting (ii) from (i), we obtain

PQ – BQ = QR – CR

⇒ PB = QC ... (iii)

In ΔτAPB and ΔτBQC

AP = BQ

[given ∠APB = ∠BQC = 90°] (each angle of a square is 90°)

PB = QC (using (iii)]

So, by using SAS congruence axiom, we have

ΔAPB ≅ ΔBQC

∴ AB = BC [c.p.c.t.]

Now, in ΔABC

AB = BC [proved above]

∴ ∠ACB = ∠BAC = x° (say) [∠s opp. to equal sides]

Also, ∠B + ∠ACB + ∠BAC = 180°

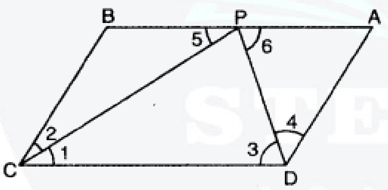
⇒ 90° + x + x = 180°

⇒ 2x° = 90°

x° = 45°

Hence, ∠BAC = 45°

5. Solution:



Since DP and CP are angle bisectors of ∠D and ∠C respectively.

: ∠1 = ∠2 and ∠3 = ∠4

Now, AB || DC and CP is a transversal

∴ ∠5 = ∠1 [alt. int. ∠s]

But ∠1 = ∠2 [given]

∴ ∠5 = ∠2

Now, in ABCP, ∠5 = ∠2

⇒ BC = BP ... (I) [sides opp. to equal ∠s of a A]

Again, AB || DC and DP is a transversal.

∴ ∠6= ∠3 (alt. int. Δs]

But ∠4 = ∠3 [given]

∴ ∠6 = ∠4

Now, in ΔADP, ∠6 = ∠4

⇒ DA = AP .... (ii) (sides opp. to equal ∠s of a A]

Also, BC = DA... (iii) (opp. sides of parallelogram)

From (i), (ii) and (iii), we have

BP = AP

Hence, P is the mid-point of side AB.

Assertion and Reason Answers-

1. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. (c) Assertion is correct statement but reason is wrong statement.

Case Study Answer:

1.

(i)	(b)	30cm
(ii)	(c)	135, 135
(iii)	(c)	Trapezium has 1 pair of parallel sides, and parallelogram has 2 pairs of parallel sides.
(iv)	(d)	equal.
(v)	(a)	BF = FC

2.

(i)	(b)	8cm
(ii)	(a)	$24cm^2$
(iii)	(c)	360°
(iv)	(c)	Triangles
(v)	(c)	Equal to 90°

Area of Parallelograms and Triangles 9

Introduction to Planar region and Area

The part of the plane enclosed by a simple closed figure is called a **planar region** corresponding to that figure. The magnitude or measure of that planar region is called its **area**.

Congruent figures and their areas

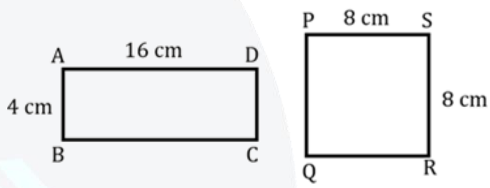
- Two figures are called congruent, if they have the same shape and the same size.
- If two figures A and B are congruent, they must have equal areas.
- Two figures having equal areas need not be congruent. In the figure,

  Area of rectangle ABCD = 16 × 4 = 64 cm^2

  Area of square PQRS = 82 = 64 cm^2

  Area of rectangle ABCD = Area of square PQRS

  But rectangle ABCD and square PQRS are not congruent.



Area of a figure

Area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

Area of the planar region

If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then ar(T) = ar(P) + ar(Q).

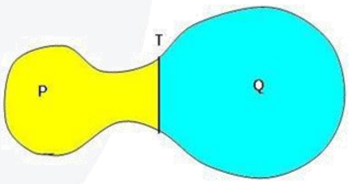
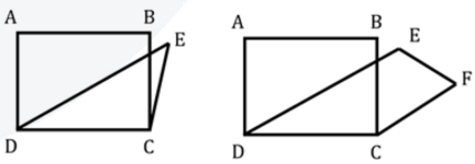
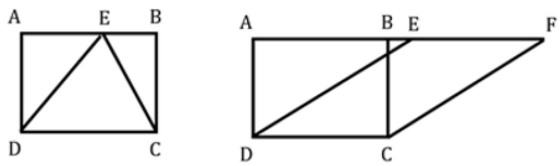


Figure on the same base and between the same parallels

- Two figures are said to be on the same base and between the same parallels if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.



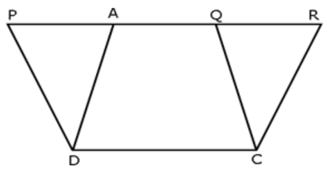
- On the same base but not between the same parallel.
- On the same base CD and between the same AF and CD
- Please note that out of the two parallels, one must be the line containing the common base.



Areas of figures on the same base and between the same parallels

- Parallelograms on the same base and between the same parallels are equal in area.

  In the figure, parallelograms PQCD and ARCD lie on the same base CD and between same parallels CD and PR. So, ar(PQCD) = ar(ARCD).

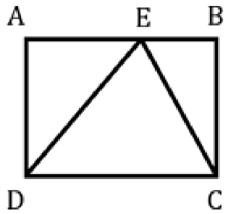


- Area of a parallelogram is the product of its any side and the corresponding altitude.
- Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.

- If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half of the area of the parallelogram.

  In the figure, triangle DEC and parallelogram ABCD are on the same CD and between the same parallels AB and CD.

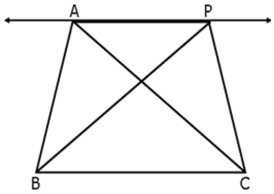
  Therefore, area of triangle DEC = ½ × area of parallelogram ABCD.



- Two triangles on the same base (or equal base) and between the same parallel are equal in area.

  In the figure, triangles ABC and PBC lie on the same base BC and between same parallels BC and AP.

  Therefore, ar(triangle ABC) = ar(triangle PBC).

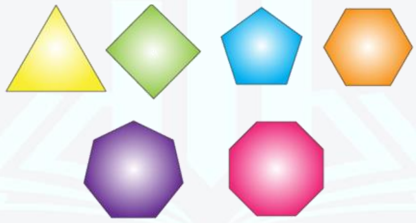


- **Area of a triangle** is half the product of its base (or any side) and the corresponding altitude (or height).

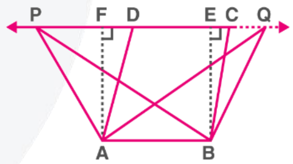
Important facts about triangles on the same base

- Two triangles with same base (or equal bases) and equal areas will have equal corresponding altitudes.
- Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.
- A median of a triangle divides it into triangles of equal areas.

The area represents the amount of planar surface being covered by a closed geometric figure.



Figures on the Common Base and Between the Same Parallels

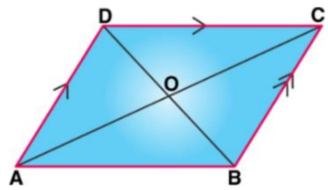


Two shapes are stated to be on the common base and between the same parallels if:

- They have a common side.
- The sides parallel to the common base and vertices opposite the common side lie on the same straight line parallel to the base.

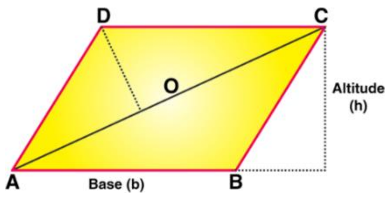
For example Parallelogram ABCD, Rectangle ABEF and Triangles ABP and ABQ

Area of a parallelogram



The area of a parallelogram is the region bounded by the parallelogram in a given two-dimension space. To recall, a parallelogram is a special type of quadrilateral which has four sides and the pair of opposite sides are parallel. In a parallelogram, the opposite sides are of equal length and opposite angles are of equal measures. Since the rectangle and the parallelogram have similar properties, the area of the rectangle is equal to the area of a parallelogram.

Area of Parallelogram Formula

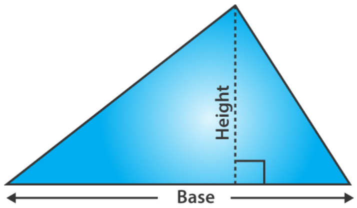


To find the area of the parallelogram, multiply the base of the perpendicular by its height. It should be noted that the base and the height of the parallelogram are perpendicular to each other, whereas the lateral side of the parallelogram is not perpendicular to the base. Thus, a dotted line is drawn to represent the height.

Area of a parallelogram = b × h

Where 'b' is the base and 'h' is the corresponding altitude (Height).

Area of a triangle



Area of a Triangle Formula

The area of the triangle is given by the formula mentioned below:

Area of a Triangle = A = ½ (b × h) square units

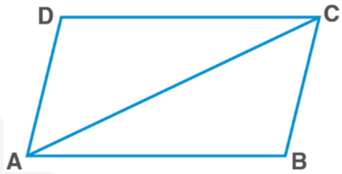
where b and h are the base and height of the triangle, respectively.

Aera = $\frac{1}{2}$ × base × perpendicular height

Now, let's see how to calculate the area of a triangle using the given formula.

The area formulas for all the different types of triangles like an area of an equilateral triangle, right-angled triangle, an isosceles triangle are given below. Also, how to find the area of a triangle with 3 sides using Heron's formula with examples.

Area of a Right-Angled Triangle



A right-angled triangle, also called a right triangle has one angle at 90° and the other two acute angles sums to 90°. Therefore, the height of the triangle will be the length of the perpendicular side.

In ΔABC and ΔCDA,

AB = CD [Opposite sides of parallelogram]

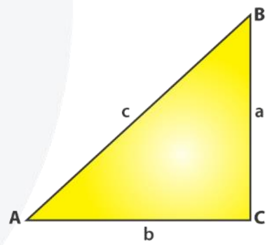
BC = AD [Opposite sides of parallelogram]

AC = AC [Common side]

ΔABC ≅ ΔCDA [by SSS rule]

Hence, proved.

Diagonals of a rhombus bisect each other at right angles



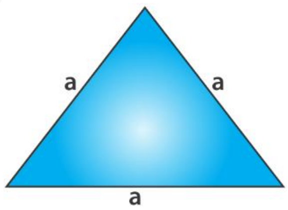
Diagonals of a rhombus bisect each – other at right angles

Area of a Right Triangle = A = ½ × Base × Height (Perpendicular distance)

From the above figure,

Area of triangle ACB = ½ ab

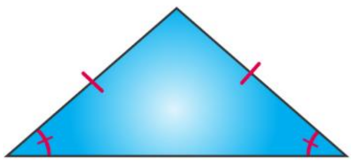
Area of an Equilateral Triangle



An equilateral triangle is a triangle where all the sides are equal. The perpendicular drawn from the vertex of the triangle to the base divides the base into two equal parts. To calculate the area of the equilateral triangle, we have to know the measurement of its sides.

Area of an Equilateral Triangle = A = ($\sqrt{3}$)/4 × side^2

Area of an Isosceles Triangle



An isosceles triangle has two of its sides equal and also the angles opposite the equal sides are equal.

Area of an Isosceles Triangle = $\frac{1}{4}b\sqrt{(4a^2 - b^2)}$

Perimeter of a Triangle

The perimeter of a triangle is the distance covered around the triangle and is calculated by adding all three sides of a triangle.

The perimeter of a triangle = P = (a + b + c) units

where a, b and c are the sides of the triangle.

Area of Triangle with Three Sides (Heron's Formula)

The area of a triangle with 3 sides of different measures can be found using Heron's formula. Heron's formula includes two important steps. The first step is to find the semi perimeter of a triangle by adding all the three sides of a triangle and dividing it by 2. The next step is that, apply the semi-perimeter of triangle value in the main formula called "Heron's Formula" to find the area of a triangle.

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

where, s is semi-perimeter of the triangle = s = (a + b + c) / 2

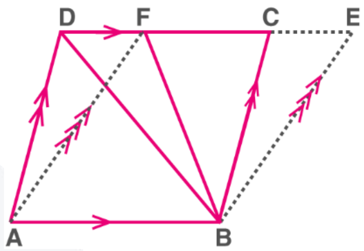
We have seen that the area of special triangles could be obtained using the triangle formula. However, for a triangle with the sides being given, the calculation of height would not be simple. For the same reason, we rely on Heron's Formula to calculate the area of the triangles with unequal lengths.

Theorems

Parallelograms on the Common Base and Between the Same Parallels

Two parallelograms are said to be on the common/same base and between the same parallels if

- They have a common side.
- The sides parallel to the common side lie on the same straight line.



Parallelogram ABCD and ABEF

Theorem: Parallelograms that lie on the common base and between the same parallels are said to have equal in area.

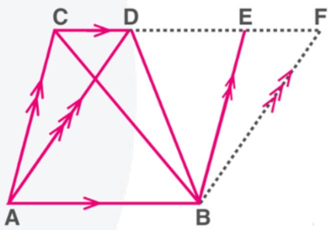
Here, ar (parallelogram ABCD) = ar(parallelogram ABEF)

Triangles on the Common Base and Between the Same Parallels

Two triangles are said to be on the common base and between the same parallels if

They have a common side.

The vertices opposite the common side lie on a straight line parallel to the common side.



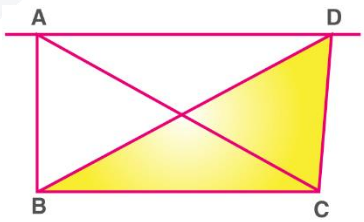
Triangles ABC and ABD

Theorem: Triangles that lie on the same or the common base and also between the same parallels are said to have an equal area.

Here, ar (ΔABC) = ar (ΔABD)

Two Triangles Having the Common Base & Equal Areas

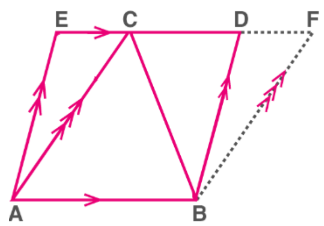
If two triangles have equal bases and are equal in area, then their corresponding altitudes are equal.



A Parallelogram and a Triangle Between the Same parallels

A triangle and a parallelogram are said to be on the same base and between the same parallels if

- They have a common side.
- The vertices opposite the common side lie on a straight line parallel to the common side.



A triangle ABC and a parallelogram ABDE

Theorem: If a triangle and a parallelogram are on the common base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

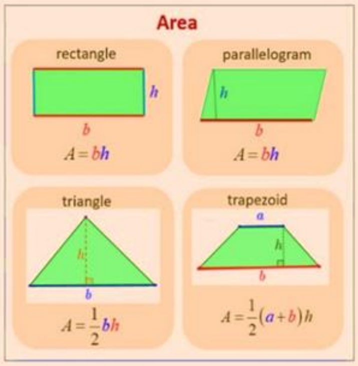
Here, ar (ΔABC) = (1/2) ar (parallelogram ABDE)

Class : 9th mathematics
Chapter- 9: Areas of Parallelograms and Triangles

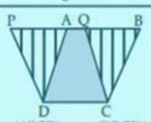
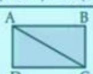
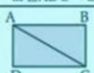
Areas of Parallelograms and Triangles

Area

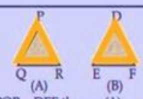
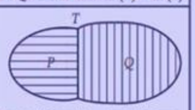
Number (in some unit) associated with part of the plane enclosed by the figure



Theorems

Statement	Figure
1. Parallelograms on the same base and between same parallels are equal in area.	 ar (ABCD) = ar (PQCD)
2. Two triangles on the same base (or equal bases) and between the same parallels are equal in area	 ar ΔADC = ar ΔABC
3. Two triangles having the same base (or equal bases) and equal areas lie between the same parallels	 if ar ΔADC = ar ΔABC then AB || CD

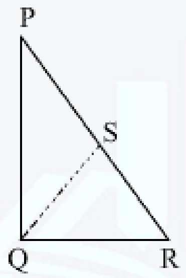
Properties

(i) If *A* and *B* are congruent figures,	ar (*A*) = ar (*B*)	 if PQR ≅ DEF then ar (A) = ar (B)
(ii) If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures *P* and *Q*	ar (*T*) = ar(*P*) + ar (*Q*)	 Area of figure *T* = Area of figure *P* + Area of figure *Q*

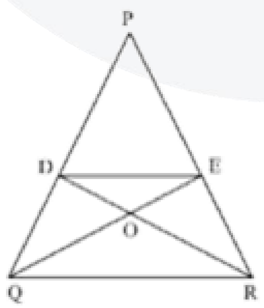
Important Questions

Multiple Choice Questions

1. What is the area of a parallelogram?
   (a) $\frac{1}{2}$ × Base × Altitude
   (b) Base × Altitude
   (c) $\frac{1}{4}$ × Base × Median
   (d) Base × Base
2. AE is a median to side BC of triangle ABC. If area(ΔABC) = 24cm, then area(ΔABE) =
   (a) 8cm
   (b) 12cm
   (c) 16cm
   (d) 18cm
3. In the figure, ∠PQR = 90°, PS = RS, QP = 12cm and QS = 6.5cm. The area of ΔPQR is



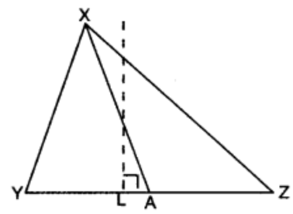
   (a) $30cm^2$
   (b) $20cm^2$
   (c) $39cm^2$
   (d) $60cm^2$
4. BCD is quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD
   (a) Is a rectangles
   (b) Is a parallelogram
   (c) Is a rhombus
   (d) Need not be any of (a), (b) or (c).
5. In ΔPQR, if D and E are points on PQ and PR respectively such that DE || QR, then ar (PQE) is equal to



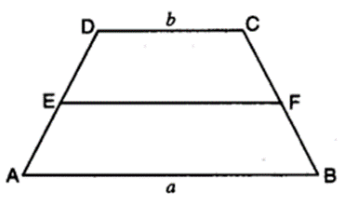
   (a) ar (PRD)
   (b) ar (DQM)
   (c) ar (PED)
   (d) ar (DQR)
6. If Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Then,
   (a) ar (AOD) = ar (BOC)
   (b) ar (AOD) > ar (BOC)
   (c) ar (AOD) < ar (BOC)
   (d) None of the above
7. For two figures to be on the same base and between the same parallels, one of the lines must be.
   (a) Making an acute angle to the common base
   (b) The line containing the common base
   (c) Perpendicular to the common base
   (d) Making an obtuse angle to the common base
8. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is:
   (a) 1 : 3
   (b) 1 : 2
   (c) 2 : 1
   (d) 1 : 1
9. If P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD, then:
   (a) ar (APB) > ar(BQC)
   (b) ar (APB) < ar(BQC)
   (c) ar (APB) = ar(BQC)
   (d) None of the above
10. A triangle and a rhombus are on the same base and between the same parallels. Then the ratio of area of triangle to that rhombus is:
    (a) 1 : 3
    (b) 1 : 2
    (c) 1 : 1
    (d) 1 : 4

Very Short Questions:

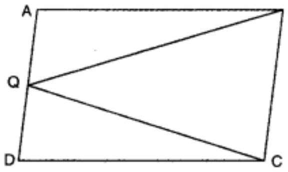
1. Two parallelograms are on equal bases and between the same parallels. Find the ratio of their areas.
2. In ΔXYZ, XA is a median on side YZ. Find ratio of ar(ΔXYA) : ar(ΔXZA)



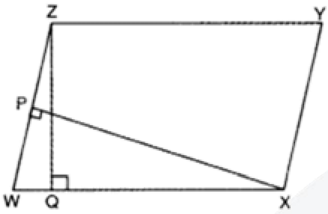
3. ABCD is a trapezium with parallel sides AB = a cm and DC = b cm (fig.). E and F are the mid-points of the non parallel sides. Find the ratio of ar(ABFE) and ar(EFCD).



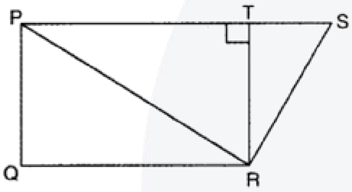
4. ABCD is a parallelogram and Q is any point on side AD. If ar(ΔQBC) = 10 cm2, find ar(ΔQAB) + ar(ΔQDC).



5. WXYZ is a parallelogram with XP ⊥ WZ and ZQ ⊥ WX. If WX = 8 cm, XP = 8 cm and ZQ = 2 cm, find YX.

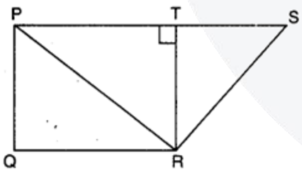


6. In figure, TR ⊥ PS, PQ || TR and PS || QR. If QR = 8 cm, PQ = 3 cm and SP = 12 cm, find ar(quad. PQRS).



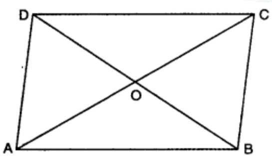
7. In the given figure, ABCD is a parallelogram and L is the mid-point of DC. If ar(quad. ABCL) is 72 cm, then find ar(ΔADC).

8. In figure, TR ⊥ PS, PQ || TR and PS || QR. If QR = 8 cm, PQ = 3 cm and SP = 12 cm, find ar (PQRS).

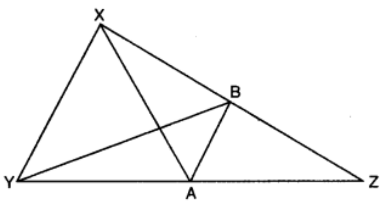


Short Questions:

1. ABCD is a parallelogram and O is the point of intersection of its diagonals. If ar(A AOD) = 4 cm\(2\) find area of parallelogram ABCD.

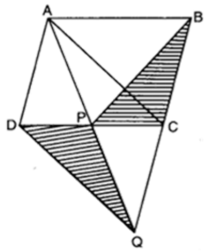


2. In the given figure of ΔXYZ, XA is a median and AB || YX. Show that YB is also a median.

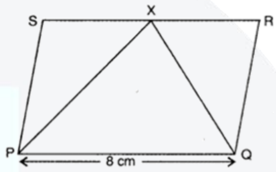


3. ABCD is a trapezium. Diagonals AC and BD intersect each other at O. Find the ratio ar (ΔAOD) : ar (ΔBOC).

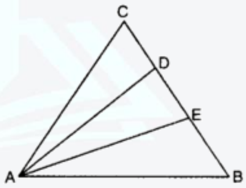
4. ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ (fig.). If AQ intersects DC at P, show that ar(ΔBPC) = ar(ΔDPQ).



5. In the figure, PQRS is a parallelogram with PQ = 8 cm and ar(ΔPXQ) = 32 cm^2. Find the altitude of gm PQRS and hence its area.

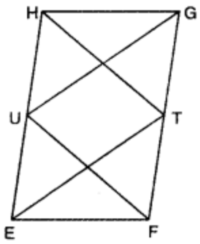


6. In ΔABC. D and E are points on side BC such that CD = DE = EB. If ar(ΔABC) = 27cm, find ar(ΔADE)



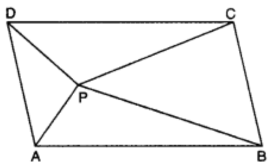
Long Questions:

1. EFGH is a parallelogram and U and T are points on sides EH and GF respectively. If ar(ΔEHT) = 16cm, find ar(ΔGUF).

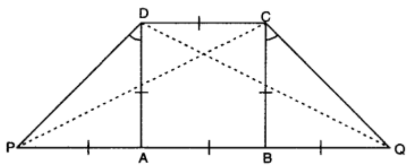


2. ABCD is a parallelogram and P is any point in its interior. Show that:

ar(ΔAPB) + ar(ΔCPD) = ar(ΔBPC) + ar(ΔAPD)

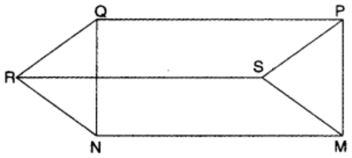


3. In the given figure, ABCD is a square. Side AB is produced to points P and Q in such a way that PA = AB = BQ. Prove that DQ = CP.



4. In the given figure, PQRS, SRNM and PQNM are parallelograms, Show that :

ar(ΔPSM) = ar(ΔQRN).



5. Naveen was having a plot in the shape of a quadrilateral. He decided to donate some portion of it to construct a home for orphan girls. Further he decided to buy a land in lieu of his donated portion of his plot so as to form a triangle.
   (i) Explain how this proposal will be implemented?
   (ii) Which mathematical concept is used in it?
   (iii) What values are depicted by Naveen?

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The area of a parallelogram and a rectangle having a common base and between same parallels are equal.

   Reason: Another name of a rectangle is a parallelogram.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The parallelogram on the same and between the same parallel are equal in area.

   Reason: The areas of parallelogram between the same parallels are equal.

Answer Key

Multiple Choice Questions

1. (b) Base × Altitude
2. (b) 12cm
3. (c) $30cm^2$
4. (d) Need not be any of (a), (b) or (c).
5. (a) ar (PRD)
6. (a) ar (AOD) = ar (BOC)
7. (b) The line containing the common base
8. (d) 1 : 1
9. (c) ar(APB) = ar(BQC)
10. (b) 1 : 2

Very Short Answer:

1. 1 : 1 [∵ Two parallelograms on the equal bases and between the same parallels are equal in area.]

2. Here, XA is the median on side YZ.

   ∴ YA = AZ

   Draw XL ⊥ YZ

   $\therefore ar(\Delta XYA) = \frac{1}{2} \times YA \times XL$

   $ar(\Delta XZA) = \frac{1}{2} \times AZ \times XL$

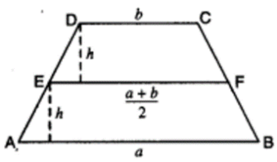
   Thus, ar(ΔXYA) : ar(ΔXZA)

   $= \frac{1}{2} \times YA \times XL : \frac{1}{2} \times AZ \times XL$

   $= 1 : 1$ [∵ YA = AZ]

3. Clearly,

   $EF = \frac{AB + DC}{2} = \frac{a+b}{2}$

   Let h be the height, then

ar [Trap. ABFE) : ar(Trap. EFCD)

$$\Rightarrow \frac{1}{2}\left[a+\left(\frac{a+b}{2}\right)\right]\times h : \frac{1}{2}\left[b+\left(\frac{a+b}{2}\right)\right]\times h$$

$$\Rightarrow \frac{2a+a+b}{2} : \frac{2b+a+b}{2}$$

⇒ 3a + b : 3b + a

4. Here, ΔQBC and parallelogram ABCD are on the same base BC and lie between the same parallels BC || AD.

∴ ar(||gm ABCD) = 2 ar(ΔQBC) ar(ΔQAB) + ar(ΔQDC) + ar(ΔQBC) = 2 ar(ΔQBC)

ar(ΔQAB) + ar(ΔQDC) = ar(ΔQBC)

Hence, ar(ΔQAB) + ar(ΔQDC) = 10cm2

[∵ ar(ΔQBC) = 10 cm2 (given)]

5. ar(||gm WXYZ) = ar(||gm WXYZ)

WX × ZQ = WZ × XP

8 × 2 = WZ × 8

⇒ WZ = 2cm

Now, YX = WZ = 2cm

[∵ opposite sides of parallelogram are equal]

6. Here,

PS || QR [given]

∴ PQRS is a trapezium

Now, TR ⊥ PS and PQ || TR [given]

⇒ PQ ⊥ PS

∴ PQ = TR = 3cm [given]

Now, ar(quad. PQRS) = $\frac{1}{2}$ (PS + QR) × PQ =

$\frac{1}{2}$(12 + 8) × 3 = 30cm^2

7. In ||gm ABCD, AC is the diagonal

∴ ar(ΔABC) = ar(ΔACD) = $\frac{1}{2}$ar(||^{gm} ABCD)

In ΔABC, AL is the median

∴ ar(ΔADL) = ar(ΔACL) = $\frac{1}{2}$ar(ΔADC)

= $\frac{1}{4}$ ar (||^{gm} ABCD)

Now, ar(quad. ABCL) = ar(ΔABC) + ar(ΔACL) =

= $\frac{3}{4}$ ar (||^{gm} ABCD)

72 × $\frac{3}{4}$ = ar (||^{gm} ABCD)

⇒ ar (||^{gm} ABCD) = 96 cm^2

∴ ar(ΔADC) = $\frac{1}{2}$ar(||^{gm} ABCD) = $\frac{1}{2}$ × 96 = 48 cm^2

8. Here, PS || QR

∴ PQRS is a trapezium in which PQ = 3cm, QR = 8cm and SP = 12cm

Now, TR I PS and PQ || TR

∴ PQRT is a rectangle

[∵ PQ || TR, PT || QR and ∠PTR = 90°]

⇒ PQ = TR = 3cm

Now, ar(PQRS) = $\frac{1}{2}$ (PS + QR) × TR

= $\frac{1}{2}$ (12 + 8) × 3 = 30 cm^2

Short Answer:

1. **Solution:**

Here, ABCD is a parallelogram in which its diagonals AC and BD intersect each other in O.

∴ O is the mid-point of AC as well as BD.

Now, in ΔADB, AO is its median

∴ ar(ΔADB) = 2 ar(ΔAOD)

[∵ median divides a triangle into two triangles of equal areas]

So, ar(ΔADB) = 2 × 4 = 8 cm^2

Now, ΔADB and ||gm ABCD lie on the same base AB and lie between same parallels AB and CD

∴ ar(ABCD) = 2 ar(ΔADB).

= 2 × 8 = 16 cm^2

2. **Solution:**

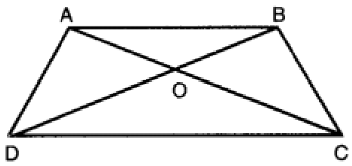
Here, in ΔXYZ, AB || YX and XA is a median.

∴ A is the mid-point of YZ. Now, AB is a line segment from mid-point of one side (YZ) and parallel to another side (AB || YX), therefore, it bisects the third side XZ.

⇒ B is the mid-point of XZ.

Hence, YB is also a median of ΔXYZ.

3. **Solution:**



Here, ABCD is a trapezium in which diagonals AC and BD intersect each other at O. ΔADC and ΔBCD are on the same base DC and between the same 'parallels i.e., AB || DC.

∴ ar(ΔADC) = ar(ΔBCD)

⇒ ar(ΔAOD) + ar(ΔODC)

= ar(ΔBOC) + ar (ΔODC)

⇒ ar(ΔAOD) = ar(ΔBOC)

$$\Rightarrow \frac{ar(\Delta AOD)}{ar(\Delta BOC)} = 1$$

4. **Solution:**

In ||^{gm} ABCD,

ar(ΔAPC) = ar(ΔBCP) ...(i)

[∵ triangles on the same base and between the same parallels have equal area]

Similarly, ar(ΔADQ) = ar(ΔADC) i ...(ii)

Now, ar(ΔADQ)–ar(ΔADP) = ar(ΔADC)–ar(ΔADP)

ar(ΔDPQ) = ar(ΔACP) ... (iii)

From (i) and (iii), we have

ar(ΔBCP) = ar(ΔDPQ)

or ar(ΔBPC) = ar(ΔDPQ)

5. **Solution:**

Since parallelogram PQRS and APXQ are on the same base PQ and lie between the same parallels PQ || SR

Now, $\frac{1}{2} \times 8 \times$ altitude = 32

altitude = 8cm

ar(||^{gm} PQRS) = 2 ar(ΔPXQ)

= 2 × 32 = 64cm^2

Hence, the altitude of parallelogram PQRS is 8cm and its area is 64cm^2.

6. **Solution:**

Since in ΔAEC, CD = DE, AD is a median.

∴ ar(ΔACD) = ar(ΔADE)

[∵ median divides a triangle into two triangles of equal areas]

Now, in ΔABD, DE = EB, AE is a median

ar(ΔADE) = ar(ΔAEB) ... (ii)

From (i), (ii), we obtain

ar(ΔACD) = ar(ΔADE) = ar(ΔAEB) $\frac{1}{3}$ ar(ΔABC)

∴ ar(ΔADE) = $\frac{1}{3} \times 27 = 9$cm^2

Long Answer:

1. **Solution:**

∴ ar(ΔEHT) = $\frac{1}{2}$ ar(||^{gm} EFGH) .....(i)

Similarly, ΔGUF and parallelogram EFGH are on the same base GF and lie between the same parallels GF and HE

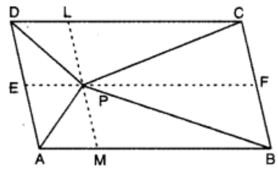
∴ ar(ΔGUF) = $\frac{1}{2}$ ar(||^{gm} EFGH) .....(ii)

From (i) and (ii), we have

ar(ΔGUF) = ar(ΔEHT)

= 16cm^2 [∵ ar(ΔEHT) = 16cm^2] [given]

2. **Solution:**



Through P, draw a line LM || DA and EF || AB

Since ΔAPB and ||gm ABFE are on the same base AB and lie between the same parallels AB and EF.

∴ ar(ΔAPB) = $\frac{1}{2}$ ar(||^{gm} ABFE) .....(i)

Similarly, ACPD and parallelogram DCFE are on the same base DC and between the same parallels DC and EF.

∴ ar(ΔCPD) = $\frac{1}{2}$ ar(||^{gm} EFGH) .....(ii)

= $\frac{1}{2}$ ar (||^{gm} ABFE) + ar(||^{gm} DCFE)

= $\frac{1}{2}$ ar (||^{gm} ABCD) .....(iii)

Since ΔAPD and parallelogram ADLM are on the same base AB and between the same parallels AD and ML

∴ ar(ΔAPD) = $\frac{1}{2}$ ar (||^{gm} ADLM) .....(iv)

Similarly, ar(ΔBPC) = $\frac{1}{2}$ arc (||^{gm} BCLM) .....(v)

Adding (iv) and (v), we have

ar(ΔAPD) + ar (ΔBPC) = $\frac{1}{2}$ ar (||^{gm} ABCD) .....(vi)

From (iii) and (vi), we obtain

ar(ΔAPB) + ar(ΔCPD) = ar(ΔAPD) + ar(ABPC)

3. **Solution:**

In ΔPAD, ∠A = 90° and DA = PA = AB

⇒ ∠ADP = ∠APD = $\frac{90°}{2}$ = 45°

Similarly, in ΔQBC, ∠B = 90° and BQ = BC = AB

⇒ ∠BCQ = ∠BQC = $\frac{90°}{2}$ = 45°

In ΔPAD and ΔQBC, we have

PA = BQ [given]

∠A = ∠B [each = 90°]

AD = BC [sides of a square]

⇒ ∠PAD ≅ ΔQBC [by SAS congruence rule]

⇒ PD = QC [c.p.c.t.]

Now, in APDC and ΔQCD

DC = DC [common]

PD = QC [prove above]

∠PDC = ∠QCD [each = 90° + 45° = 135°]

⇒ ΔPDC = ΔQCD [by SAS congruence rule]

⇒ PC = QD or DQ = CP

4. Solution:

Since PQRS is a parallelogram.

∴ PS = QR and PS || QR

Since SRNM is also a parallelogram.

∴ SM = RN and SM || RN

Also, PQNM is a parallelogram

∴ PM || QM and PM = QM

Now, in APSM and ΔQRN

PS = QR

SM = RN

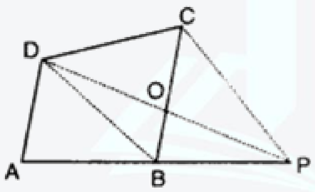
PM = QN

ΔPSM ≅ ΔQRN [by SSS congruence axiom]

∴ ar (ΔPSM) = ar (ΔQRN) [congruent triangles have same areas)

5. Solution:

(i) Let ABCD be the plot and Naveen decided to donate some portion to construct a home for orphan girls from one corner say C of plot ABCD. Now,



Naveen also purchases equal amount of land in lieu of land CDO, so that he may have triangular form of plot. BD is joined. Draw a line through C parallel to DB to meet AB produced in P.

Join DP to intersect BC at 0.

Now, ABCD and ABPD are on the same base and between same parallels CP || DB.

ar(ΔBCD) = ar(ΔBPD) ar(ΔCOD) + ar(ΔDBO) = ar(ΔBOP) + ar(ΔDBO)

ar(ACOD) = ar(ABOP) ar(quad. ABCD)

= ar(quad. ABOD) + ar(ΔCOD)

= ar(quad. ABOD) + ar(ΔBOP)

[∵ ar(ACOD) = ar(ABOP)] (proved above]

= ar(ΔAPD)

Hence, Naveen purchased the portion ABOP to meet his requirement.

(ii) Two triangles on the same base and between same parallels are equal in area.

(iii) We should help the orphan children.

Assertion and Reason Answers-

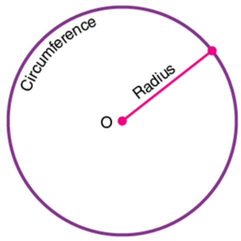
1. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. (c) Assertion is correct statement but reason is wrong statement.

Circles 10

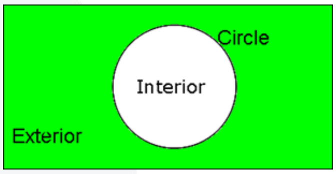
Circle

- The set of all the points in a plane that is at a fixed distance from a fixed point makes a circle.
- A Fixed point from which the set of points are at fixed distance is called the centre of the circle.
- A circle divides the plane into 3 parts: interior (inside the circle), the circle itself and exterior (outside the circle)



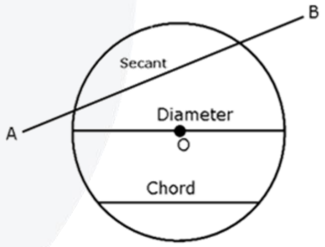
Division of a plane using circle

- A circle divides the plane on which it lies into three parts: inside the circle, the circle and outside the circle.
- All the points lying inside a circle are called its **interior points** and all those points which lie outside the circle are called its **exterior points.**
- The collection (set) of all interior points of a circle is called the **interior of the circle** while the collection (set) of all exterior points of a circle is called the **exterior of the circle.**



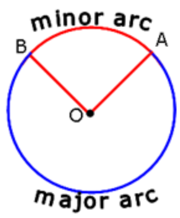
Chord, diameter and secant of a circle

- A line can meet a circle at the most in two points and the line segment joining two points on a circle is called a **chord** of the circle.
- A chord passing through the center of the circle is called a **diameter** of the circle. A diameter of the circle is its longest chord. It is equal to two times the radius.
- A line which meets a circle in two points is called a **secant** of the circle.



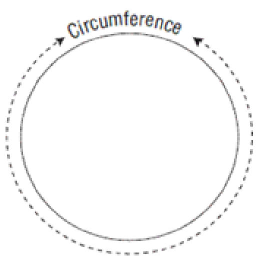
Arc of the circle

- A (continuous) part of a circle is called an arc of the circle. The **arc** of a circle is denoted by the symbol ' '.
- When an arc is formed, it divides the circle into two pieces (between the points A and B), the smaller one and the longer one. The smaller one is called the **minor arc** of the circle, and the greater one is called the **major arc** of the circle.

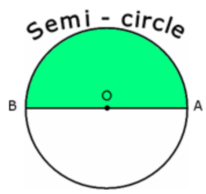


Circumference and Semi-circle

- The length of the complete circle is called the **circumference** of the circle.



- One-half of the whole arc (circumference) of a circle is called a **semi-circle.**



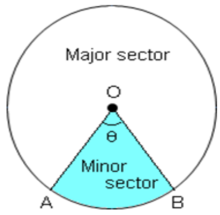
Central angle and Degree measure

- Any angle whose vertex is centre of the circle is called a **central angle**.
- The **degree measure of a minor arc** is the measure of the central angle subtended by an arc.
- The degree measure of a circle is 360°. The degree measure of a semi-circle is 180° (half of the circle).
- The degree measure of a major arc is (360° - θ°), where θ° is the degree measure of the corresponding minor arc.

Congruent circles and arcs

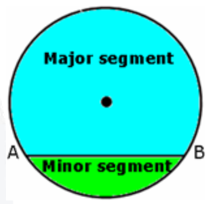
- Two circles are said to be **congruent** if and only if either of them can be superposed on the other so as to cover it exactly.
- Accordingly, two **arcs** of a circle (or of congruent circles) are **congruent** if either of them can be superposed on the other so as to cover it exactly.

Sector of a circle



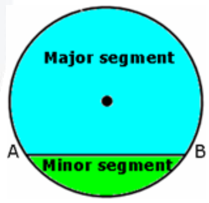
- The part of the plane region enclosed by an arc of a circle and its two bounding radii is called **sector** of a circle.
- If the central angle of a sector is more than180°, then the sector is called a **major sector** and if the central angle is less than 180°, then the sector is called a **minor sector**.

Segment of a circle



- A chord of a circle divides it into two parts. Each part is called a **segment** of the circle.
- The part containing the minor arc is called the **minor segment**, and the part containing the major arc is called the **major segment**.

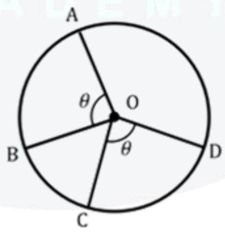
Segment of a circle



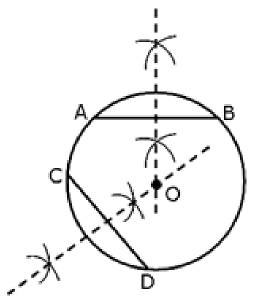
- A chord of a circle divides it into two parts. Each part is called a **segment** of the circle.
- The part containing the minor arc is called the **minor segment**, and the part containing the major arc is called the **major segment**.

Angle subtended by a chord and perpendicular drawn to a chord

- Equal chords of a circle subtend equal angles at the centre.

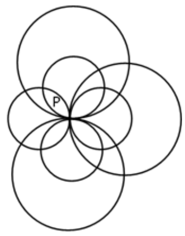


- If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.
- In a circle, perpendicular from the center to a chord bisects the chord.
- A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
- Perpendicular bisectors of two chords of a circle, intersect each other at the centre of the circle.

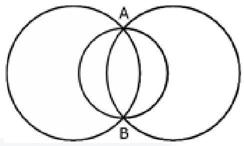


Number of circle through one or more point(s)

- An infinite number of circles can be drawn through a given point, say P.



- An infinite number of circles can be drawn through two given points, say A and B.



- One and only one circle can be drawn through three non-collinear points.

Distance of chord from the centre

- The length of the perpendicular from a point to a line is the distance of the line from the point.
- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).
- Chords equidistant from the centre of a circle are equal in length.

Angle subtended by an Arc of a circle

- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- If two chords of a circle are equal, then their corresponding arcs are congruent.
- Conversely, if two arcs are congruent, then their corresponding chords are equal.
- Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

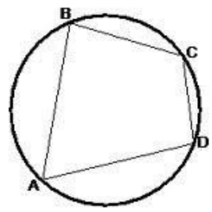
Con-cyclic points

- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points are con-cyclic, i.e., they lie on the same circle.
- Angles in the same segment of a circle are equal.

Angle in a semi-circle

- An angle in a semi-circle is a right angle.
- The arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

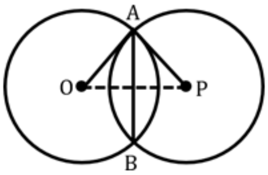
Cyclic quadrilaterals



A quadrilateral, all the four vertices of which lie on a circle is called a **cyclic quadrilateral**. The four vertices A, B, C and D are said to be concyclic points.

Properties of cyclic quadrilateral

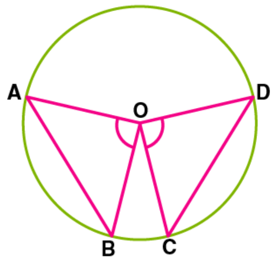
- The opposite angles of a cyclic quadrilateral are supplementary i.e. their sum is 180°.
- If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.
- Any exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- The quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.
- The line of centres of two intersecting circles subtends equal angles at the two points of intersection.



In the figure, angle OAM = angle PAM.

- If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, then it is a rectangle.
- If the non-parallel sides of a trapezium are equal, then it is cyclic.

Theorem of equal chords subtending angles at the centre.



Equal chords subtend equal angles at the centre.

Proof: AB and CD are the 2 equal chords.

In Δ AOB and Δ COD

OB = OC [Radii]

OA = OD [Radii]

AB = CD [Given]

ΔAOB ≅ ΔCOD (SSS rule)

Hence, ∠AOB = ∠COD [CPCT]

Theorem of equal angles subtended by different chords.

If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Proof: In ΔAOB and ΔCOD

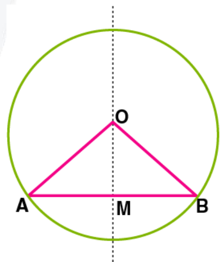
OB = OC [Radii] ∠AOB=∠COD [Given]

OA = OD [Radii]

ΔAOB ≅ ΔCOD (SAS rule)

Hence, AB=CD [CPCT]

Perpendicular from the centre to a chord bisects the chord.



Perpendicular from the centre of a circle to a chord bisects the chord.

Proof: AB is a chord and OM is the perpendicular drawn from the centre.

From ΔOMB and ΔOMA,

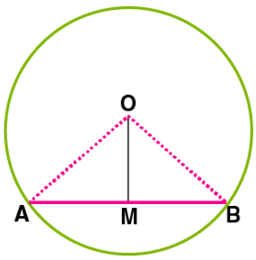
∠OMA = ∠OMB = 900 OA = OB (radii)

OM = OM (common)

Hence, ΔOMB ≅ ΔOMA (RHS rule)

Therefore AM = MB [CPCT]

A Line through the centre that bisects the chord is perpendicular to the chord.



Proof: OM drawn from the center to bisect chord AB.

From ΔOMA and ΔOMB,

OA = OB (Radii)

OM = OM (common)

AM = BM (Given)

Therefore, ΔOMA ≅ ΔOMB (SSS rule)

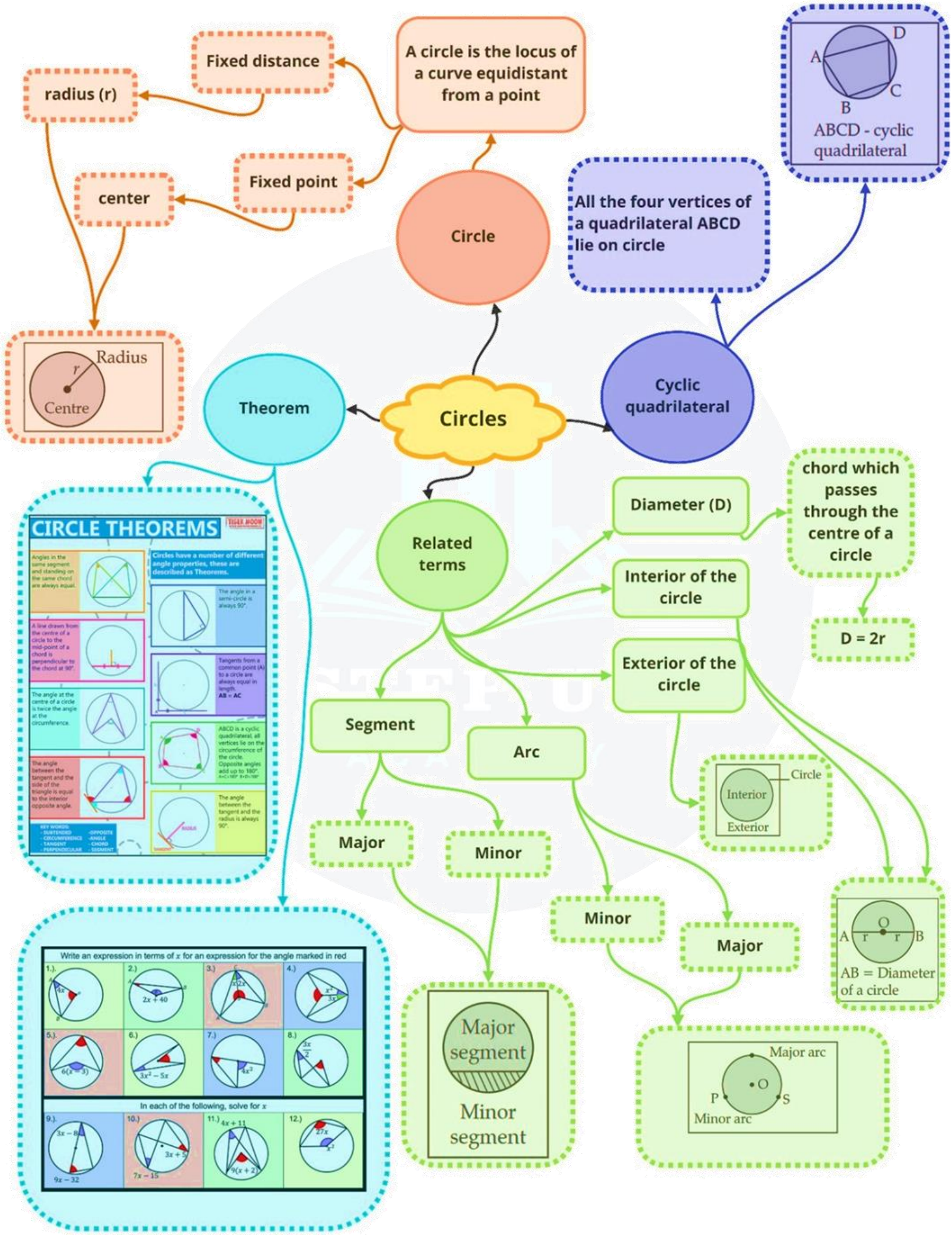
⇒ ∠OMA = ∠OMB (C.P.C.T)

But, ∠OMA + ∠OMB = 1800

Hence, ∠OMA = ∠OMB = 900 ⇒ OM⊥AB

Class : 9th mathematics

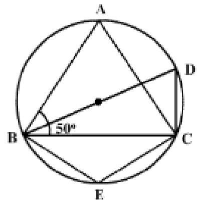
Chapter- 10: Circles



Important Questions

Multiple Choice Questions

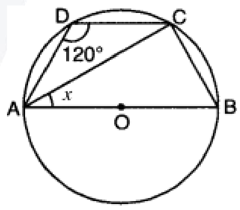
1. If there are two separate circles drawn apart from each other, then the maximum number of common points they have:
   (a) 0
   (b) 1
   (c) 2
   (d) 3
2. D is diameter of a circle and AB is a chord. If AD = 50cm, AB = 48cm, then the distance of AB from the Centre of the circle is
   (a) 6 cm
   (b) 8 cm
   (c) 5 cm
   (d) 7 cm
3. In a circle with center O and a chord BC, points D and E lie on the same side of BC. Then, if ∠BDC = 80°, then ∠BEC =
   (a) 80°
   (b) 120°
   (c) 160°
   (d) 40°
4. The center of the circle lies in______ of the circle.
   (a) Interior
   (b) Exterior
   (c) Circumference
   (d) None of the above
5. If chords AB and CD of congruent circles subtend equal angles at their centers, then:
   (a) AB = CD
   (b) AB > CD
   (c) AB < AD
   (d) None of the above
6. Segment of a circle is the region between an arc and .... of the circle.
   (a) Perpendicular
   (b) Radius
   (c) Chord
   (d) Secant
7. In the figure, triangle ABC is an isosceles triangle with AB = AC and measure of angle ABC = 50°. Then the measure of angle BDC and angle BEC will be
   (a) 60°, 100°
   (b) 80°, 100°
   (c) 50°, 100°
   (d) 40°, 120°



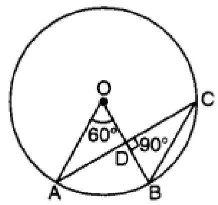
8. The region between chord and either of the arc is called.
   (a) A sector
   (b) A semicircle
   (c) A segment
   (d) A quarter circles
9. The region between an arc and the two radii joining the Centre of the end points of the arc is called a:
   (a) Segment
   (b) Semi circle
   (c) Minor arc
   (d) Sector
10. If a line intersects two concentric circles with Centre O at A, B, C and D, then:
   (a) AB = CD
   (b) AB > CD
   (c) AB < CD
   (d) None of the above

Very Short Questions:

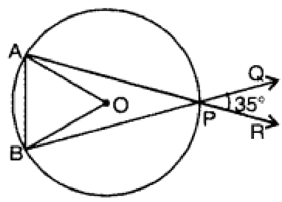
1. In the figure, O is the Centre of a circle passing through points A, B, C and D and ∠ADC = 120°. Find the value of x.



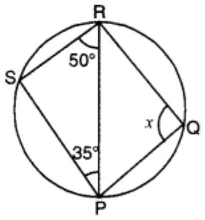
2. In the given figure, O is the Centre of the circle, ∠AOB = 60° and CDB = 90°. Find ∠OBC.



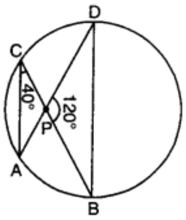
3. In the given figure, O is the Centre of the circle with chords AP and BP being produced to R and Q respectively. If ∠QPR = 35°, find the measure of ∠AOB.



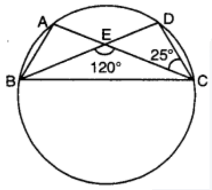
4. In the figure, PQRS is a cyclic quadrilateral. Find the value of x.



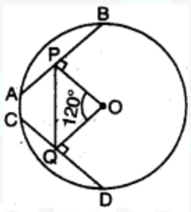
5. In the given figure, ∠ACP = 40° and BPD = 120°, then find ∠CBD.



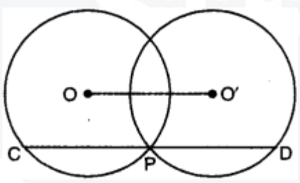
6. In the given figure, if ∠BEC = 120°, ∠DCE = 25°, then find ∠BAC.



7. In the given figure, AB and CD are two equal chords of a circle with Centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠POQ = 120°, find ∠ APQ.

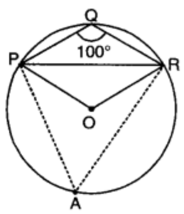


8. Two circles whose centers are O and O' intersect at P. Through P, a line parallel to OO', intersecting the circles at C and D is drawn as shown in the figure. Prove that CD = 2OO'

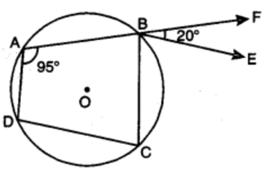


Short Questions:

1. In the given figure, PQR = 100°, where P, Q and R are points on a circle with Centre O. Find LOPR.

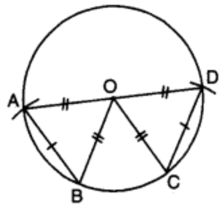


2. In figure, ABCD is a cyclic quadrilateral in which AB is extended to F and BE || DC. If ∠FBE = 20° and DAB = 95°, then find ∠ADC.

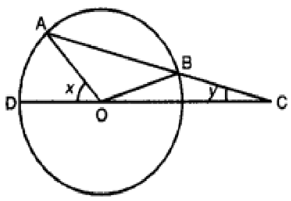


3. If the diagonals of a cyclic quadrilateral are diameters of the circle through the opposite vertices of the quadrilateral. Prove that the quadrilateral is a rectangle.

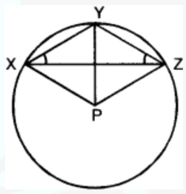
4. Equal chords of a circle subtends equal angles at the Centre.



5. In the figure, chord AB of circle with Centre O, is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If ∠ACD = y and ∠AOD = x, show that x = 3y.

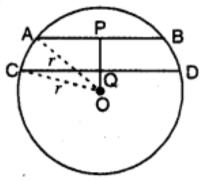


6. In the given figure, P is the Centre of the circle. Prove that: ∠XPZ = 2(∠X∠Y + ∠YXZ).

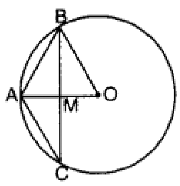


Long Questions:

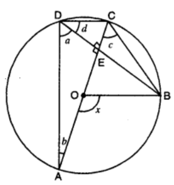
1. In the given figure, O is the Centre of a circle of radius r cm, OP and OQ are perpendiculars to AB and CD respectively and PQ = 1cm. If AB || CD, AB = 6cm and CD = 8cm, determine r



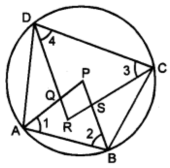
2. In a circle of radius 5cm, AB and AC are two chords such that AB = AC = 6cm, as shown in the figure. Find the length of the chord BC.



3. In the given figure, ABCD is a square. Side AB is produced to points P and Q in such a way that PA = AB = BQ. Prove that DQ = CP.



4. Show that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



5. PQ and PR are the two chords of a circle of radius r. If the perpendiculars drawn from the Centre of the circle to these chords are of lengths a and b, PQ = 2PR, then prove that:

$$b^2 = \frac{a^2}{4} + \frac{3}{4}r^2$$

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: In a circle of radius 6 cm, the angle of a sector 60°.Then the area of the sector is 186/7 cm^2.

   Reason: Area of the circle with radius r is πr^2.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

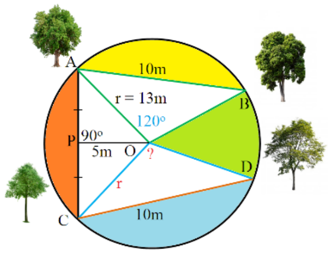
   Assertion: The length of the minute hand of a clock is 7cm. Then the area swept by the minute hand in 5 minutes is $12^5/6$ cm^2.

   Reason: The length of an arc of a sector of angle θ and radius r is given by l = θ/360 × 2πr.

Case Study Questions:

1. Read the Source/ Text given below and answer these questions:

   A farmer has a circular garden as shown in the picture above. He has a different type of trees, plants and flower plants in his garden. In the garden, there are two mango trees



   A and B at a distance of AB = 10m. Similarly, he has two Ashoka trees at the same distance of 10m as shown at C and D. AB subtends ∠AOB=120° at the center O. The perpendicular distance of AC from center is 5m. The radius of the circle is 13m.

   Now answer the following questions:

   i. What is the value of ∠COD?
      a. 60°
      b. 120°
      c. 100°
      d. 80°

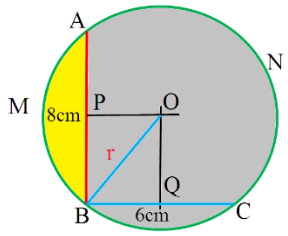
   ii. What is the distance between mango tree A and Ashok tree C?
      a. 12m
      b. 24m
      c. 13m
      d. 15m

   iii. What is the value of ∠OAB?
      a. 60°
      b. 120°
      c. 30°
      d. 90°

   iv. What is the value of ∠OCD?
      a. 30°
      b. 120°
      c. 60°
      d. 90°

   v. What is the value of ∠ODC?
      a. 90°
      b. 120°
      c. 60°
      d. 30°

2. Read the Source/ Text given below and answer any four questions:



As Class IX C' s teacher Mrs. Rashmi entered in the class, She told students to do some practice on circle chapter. She Draws two-line AB and BC so that AB = 8cm and BC = 6cm. She told all students To make this shape in their notebook and draw a circle passing through the three points A, B and C.

i. Dileep drew AB and BC as per the figure
ii. He drew perpendicular bisectors OP and OQ of the line AB and BC.
iii. OP and OQ intersect at O
iv. Now taking O as centre and OB as radius he drew The circle which passes through A, B and C.
v. He noticed that A, O and C are collinear.

Answer the following questions:

i. What you will call the line AOC?
   a. Arc
   b. Diameter
   c. Radius
   d. Chord

ii. What is the measure of ∠ABC?
   a. 60°
   b. 90°
   c. 45°
   d. 75°

iii. What you will call the yellow color shaded area AMB?
   a. Arc.
   b. Sector.
   c. Major segment.
   d. Minor Segment.

iv. What you will call the grey colour shaded area BCNA?
   a. Arc.
   b. Sector.
   c. Major segment.
   d. Minor Segment.

v. What is the radius of the circle?
   a. 4cm
   b. 3cm
   c. 7cm
   d. 5cm

Answer Key

Multiple Choice Questions

1. (a) 0
2. (d) 7cm
3. (a) 80°
4. (a) Interior
5. (a) AB = CD
6. (c) Chord
7. (b) 80°, 100°
8. (c) A segment
9. (d) Sector
10. (a) AB = CD

Very Short Answer:

1. Since ABCD is a cyclic quadrilateral

   ∠ADC + ∠ABC = 180°

   [∴ opp. ∠s of a cyclic quad. are supplementary]

   120° + ∠ABC = 180°

   ∠ABC = 180° – 120° = 60°

   Now, ∠ACB = 90° [angle in a semicircle]

   In rt. ∠ed ΔCB, ∠ACB = 90°

   ∠CAB + ∠ABC = 90°

   x + 60° = 90°

   x = 90° -60°

   x = 30°

2. Since angle subtended at the Centre by an arc is double the angle subtended at the remaining part of the circle.

   ∴ ∠ACB = 1/3 ∠AOB = 1/3 x 60° = 30°

   Now, in ACBD, by using angle sum property, we have

   ∠CBD + ∠BDC + ∠DCB = 180°

   ∠CBO + 90° + ∠ACB = 180°

   [∵ ∠CBO = ∠CBD and ∠ACB = ∠DCB are the same ∠s]

   ∠CBO + 90° + 30° = 180°

   ∠CBO = 180o – 90° – 30° = 60°

   or ∠OBC = 60°

3. ∠APB = ∠RPQ = 35° [vert. opp. ∠s]

   Now, ∠AOB and ∠APB are angles subtended by an arc AB at Centre and at the remaining part of the circle.

∴ ∠AOB = 2∠APB = 2 × 35° = 70°

4. In ΔPRS, by using angle sum property, we have

∠PSR + ∠SRP + ∠RPS = 180°

∠PSR + 50° + 35o = 180°

∠PSR = 180° – 85o = 95°

Since PQRS is a cyclic quadrilateral

∴ ∠PSR + ∠PQR = 180°

[∵ opp. ∠s of a cyclic quad. are supplementary]

95° + x = 180°

x = 180° – 95°

x = 85°

5. ∠BDP = ∠ACP = 40° [angle in same segment]

Now, in ΔBPD, we have

∠PBD + ∠BPD + ∠BDP = 180°

⇒ ∠PBD + 120° + 40° = 180°

⇒ ∠PBD = 180° – 160o = 20°

or ∠CBD = 20°

6. ∠BEC is exterior angle of ΔCDE.

∴ ∠CDE + ∠DCE = ∠BEC

⇒ ∠CDE + 25° = 120°

⇒ ∠CDE = 95°

7. Arc XY subtends ∠XPY at the Centre P and ∠XZY in the remaining part of the circle.

∴ ∠XPY = 2 (∠X∠Y)

Similarly, arc YZ subtends ∠YPZ at the Centre P and ∠YXZ in the remaining part of the circle.

∴ ∠YPZ = 2(∠YXZ) ....(ii)

Adding (i) and (ii), we have

∠XPY + ∠YPZ = 2 (∠XZY + ∠YXZ)

∠XP2 = 2 (∠XZY + ∠YXZ)

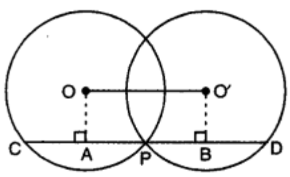
8. Draw OA ⊥ CD and O'B ⊥ CD

Now, OA ⊥ CD

OA ⊥ CP

CA = AP = 1/2 CP

CP = 2AP ....(i)



Similarly, O'B ⊥ CD

O'B ⊥ PD

⇒ PB = BD = 1/2 PD

⇒ PD = 2PB

Also, CD = CP + PD

= 2AP + 2PB = 2(AP + PB) = 2AB

CD = 2OO' [∵ OABO' is a rectangle]

Short Answer:

1. **Solution:**

Take any point A on the circumcircle of the circle.

Join AP and AR.

∵ APQR is a cyclic quadrilateral.

∴ ∠PAR + ∠PQR = 180° [sum of opposite angles of a cyclic quad. is 180°]

∠PAR + 100° = 180°

⇒ Since ∠POR and ∠PAR are the angles subtended by an arc PR at the Centre of the circle and circumcircle of the circle.

∠POR = 2∠PAR = 2 x 80° = 160°

∴ In APOR, we have OP = OR [radii of same circle]

∠OPR = ∠ORP [angles opposite to equal sides]

Now, ∠POR + ∠OPR + ∠ORP = 180°

⇒ 160° + ∠OPR + ∠OPR = 180°

⇒ 2∠OPR = 20°

⇒ ∠OPR = 10°

2. **Solution:**

Sum of opposite angles of a cyclic quadrilateral is 180°.

∴ ∠DAB + ∠BCD = 180°

⇒ 95° + ∠BCD = 180°

⇒ ∠BCD = 180° – 95° = 85°

∵ BE || DC

∴ ∠CBE = ∠BCD = 85°[alternate interior angles]

∴ ∠CBF = CBE + ∠FBE = 85° + 20° = 105°

Now, ∠ABC + 2CBF = 180° [linear pair]

and ∠ABC + ∠ADC = 180° [opposite angles of cyclic quad.]

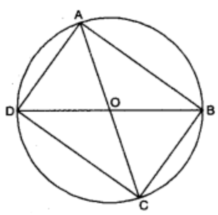
Thus, ∠ABC + ∠ADC = ∠ABC + 2CBF

⇒ ∠ADC = CBF

⇒ ∠ADC = 105° [∵ CBF = 105°]

3. **Solution:**

Here, ABCD is a cyclic quadrilateral in which AC and BD are diameters.



Since AC is a diameter.

∴ ∠ABC = ∠ADC = 90°

[∵ angle of a semicircle = 90°]

Also, BD is a diameter

∴ ∠BAD = ∠BCD = 90° [∵ angle of a semicircle = 90°]

Now, all the angles of a cyclic quadrilateral ABCD are 90 each.

Hence, ABCD is a rectangle.

4. **Solution:**

Given: In a circle C(O, r), chord AB = chord CD

To Prove: ∠AOB = ∠COD.

Proof: In ΔAOB and ΔCOD

AO = CO (radii of same circle]

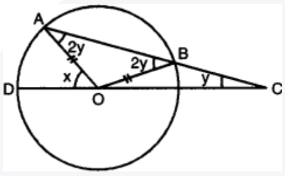
BO = DO [radii of same circle]

Chord AB = Chord CD (given]

⇒ ΔAOB = ACOD [by SSS congruence axiom]

⇒ ∠AOB = COD (c.p.c.t.]

5. **Solution:**



In AOBC, OB = BC

⇒ ∠BOC = ∠BCO = y [angles opp. to equal sides are equal]

∠OBA is the exterior angle of ΔBOC

So, ∠ABO = 2y [ext. angle is equal to the sum of int. opp. angles]

Similarly, ∠AOD is the exterior angle of ΔAOC

∴ x = 2y + y = 3y

6. **Solution:**

Arc XY subtends ∠XPY at the Centre P and ∠XZY in the remaining part of the circle.

∴ ∠XPY = 2 (∠X∠Y)

Similarly, arc YZ subtends ∠YPZ at the Centre P and ∠YXZ in the remaining part of the circle.

∴ ∠YPZ = 2(∠YXZ) ....(ii)

Adding (i) and (ii), we have

∠XPY + ∠YPZ = 2 (∠XZY + ∠YXZ)

∠XPZ = 2 (∠XZY + ∠YXZ)

Long Answer:

1. Solution:

Since the perpendicular drawn from the Centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

Consequently, AP = BP = $\frac{1}{2}$ AB = 3cm

and CQ = QD = $\frac{1}{2}$ CD = 4cm

In right-angled AQAP, we have

$OA^2 = OP^2 + AP^2$

$r^2 = OP^2 + 32$

$r^2 = OP^2 + 9$

In right-angled ΔOCQ, we have

$OC^2 = OQ^2 + CQ^2$

$r^2 = OQ^2 + 42$

$p^2 = OQ^2 + 16$ ... (ii)

From (i) and (ii), we have

$OP^2 + 9 = OQ^2 + 16$

$OP^2 - OQ^2 = 16 - 9$

$x^2 - (x - 1)^2 = 16 - 9$ [where OP = x and PQ = 1cm given]

$x^2 - y^2 - 1 + 2x = 7$

$2x = 7 + 1$

$x = 4$

⇒ OP = 4cm

From (i), we have

$r^2 = (4)^2 + 9$

$r^2 = 16 + 9 = 25$

r = 5cm

2. **Solution:**

Here, OA = OB = 5cm [radii]

AB = AC = 6cm

∴ B and C are equidistant from A.

∴ AO is the perpendicular bisector of chord BC and it intersect BC in M.

Now, in rt. ∠ed ΔAMB, M = 90° .... (i)

∴ By using Pythagoras Theorem, we have

$BM^2 = AB^2 - AM^2$

$= 36 - AM^2$

Also, in rt. ∠ed ΔBMO, ∠M = 90°

∴ By using Pythagoras Theorem, we have

$BM^2 = BO^2 - MO^2 = 25 - (AO - AM)^2$

From (i) and (ii), we obtain

$25 - (AO - AM)^2 = 36 - AM12$

$25 - AOC - AM^2 + 240 \times AM = 36 - AM12$

$25 - 25 + 2 \times 5 \times AM = 36$

10 AM = 36

AM = 3.6cm

From (i), we have

$BM^2 = 36 - (3.6)2 = 36 - 12.96 = 23.04$

BM = $\sqrt{23.04}$ = 4.8cm

Thus, BC = 2 × BM

= 2 × 4.8 = 9.6cm

Hence, the length of the chord BC is 9.6cm.

3. Solution:

Here, AC is a diameter of the circle.

∴ $\angle ADC = 90°$

⇒ $\angle a + \angle d = 90°$

In right-angled ΔAED, $\angle E = 90°$

∴ $\angle a + 2b = 90°$

From (i) and (ii), we obtain

$\angle b = \angle d$ .... (iii)

Also, $\angle a = \angle c$ ... (iv)

[∠s subtended by the same segment are equal]

Now, ∠AOB and ∠ADB are angles subtended by an arc AB at the Centre and at the remaining part of the circle.

∴ $\angle ADB = \frac{1}{2}\angle ADB \Rightarrow \angle a = \frac{x}{2}$

From (iv), we have $\angle a = \angle c = \frac{x}{2}$

Again, $\angle AOB + \angle BOC = 180°$

⇒ $\angle BOC = 180° - \angle AOB = 180° - x$

∠BOC and ∠BDC are angles subtended by an arc BC at the centre and at the remaining part of the circle.

∴ $\angle BDC = \frac{1}{2}\angle BOC$

∴ $\angle d = \frac{1}{2}(180° - x) = 90° - \frac{x}{2}$

4. Solution:

Given: A cyclic quadrilateral ABCD in which AP, BP, CR and DR are the angle bisectors of ∠A, ∠B, 2C and ∠D respectively such that a quadrilateral PQRS is formed.

To Prove: PQRS is a cyclic quadrilateral.

Proof: Since ABCD is a cyclic quadrilateral.

∴ $\angle A + 2C = 180°$ and $\angle B + \angle D = 180°$

Also, AP, BP, CR and DR are the angle bisectors of ∠A, ∠B, ∠C and ∠D respectively.

∴ $\angle 1 = \frac{1}{2}\angle A, \angle 2 = \frac{1}{2}\angle B, \angle 3 = \frac{1}{2}\angle C$ and $\angle 4 = \frac{1}{2}\angle D$

From (i), we have $\frac{1}{2}\angle A + \frac{1}{2}\angle A = \frac{1}{2}(\angle A + \angle C)$

$= \frac{1}{2} \times 180° = 90°$

and $\frac{1}{2}\angle B + \frac{1}{2}\angle D = \frac{1}{2}(\angle B + \angle D)$

$= \frac{1}{2} \times 180° = 90°$

or $\angle 1 + \angle 3 = 90°$

and $\angle 2 + \angle 4 = 90°$

Now, in ΔAPB, by angle sum property of a Δ

$\angle 1 + \angle 2 + \angle P = 180°$ ... (iii)

Again, in ΔCRD, by angle sum property of a Δ

$\angle 3 + \angle 4 + \angle R = 180°$ ...(iv)

Adding (iii) and (iv), we have

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle P + \angle R = 180° + 180°$

$90° + 90° + \angle P + \angle R = 360°$ [using (ii)]

$\angle P + \angle R = 360° - 180° = 180°$

i.e., the sum of one pair of the opposite angles of quadrilateral PQRS is 180°.

Hence, the quadrilateral PQRS is a cyclic quadrilateral.

5. Solution:

In circle Clo, r), PQ and PR are two chords, draw OM I PQ, OL I PR, such that OM = a and OL = b. Join OP. Since the perpendicular from the Centre of the circle to the chord of the circle, bisects the chord.

∴ We have, $PM = MQ = \frac{1}{2}PQ$

and $PL = LR = \frac{1}{2}PR$

In rt, ΔOMP, $\angle M = 90°$

∴ By Pythagoras Theorem, we have

$PM^2 = OP^2 - OM^2$

$\left(\frac{1}{2}PQ\right)^2 = r^2 - a^2$

$\frac{PQ^2}{4} = r^2 - a^2$

⇒ $PQ^2 = 4r^2 - 4a^2$ ....(i)

Again, in rt. ΔOLP, $\angle L = 90°$

∴ By Pythagoras Theorem, we have

$PL^2 = OP^2 - OL^2$

$\left(\frac{1}{2}PR\right)^2 = r^2 - b^2$

$\frac{PR^2}{4} = r^2 - b^2$

⇒ $PR^2 = 4r^2 - 4b^2$ ....(ii)

Also, PQ = 2PR [given]

⇒ $PQ^2 = 4PR^2$ ....(iii)

From (i), (ii) and (iii), we have

$4r^2 - 4a^2 = 4(4r^2 - 4b^2)$

$\Rightarrow r^2 - a^2 = 4r^2 - 4b^2$

$\Rightarrow 4b^2 = 4r^2 - r^2 + a^2$

$\Rightarrow 4b^2 = 3r^2 + a^2$

$\Rightarrow b^2 = \frac{3}{4}r^2 + \frac{1}{4}a^2$

or $b^2 = \frac{1}{4}a^2 + \frac{3}{4}r^2$

Assertion and Reason Answers-

1. (b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

2. (b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Case Study Answer:

1.

(i)	(b)	120°
(ii)	(b)	24m
(iii)	(c)	30°
(iv)	(a)	30°
(v)	(d)	30°

2.

(i)	(b)	Diameter
(ii)	(b)	90°
(iii)	(d)	Minor Segment.
(iv)	(c)	Major segment.
(v)	(d)	5cm

Geometric Constructions 11

Constructions

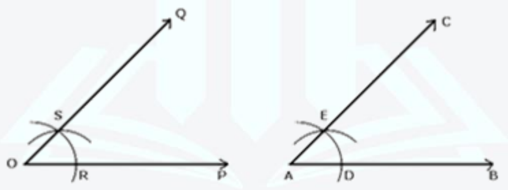
To Construct an Angle Equal to a Given Angle

Given: Any ∠POQ and a point A

Required: To construct an angle at A equal to ∠POQ

Steps of Construction:

i. With O as centre and any (suitable) radius, draw an arc to meet OP at R and OQ at S.
ii. Through A draw a line AB.
iii. Taking A as centre and same radius (as in step 1), draw an arc to meet AB at D.
iv. Measure the segment RS with compasses.
v. With D as centre and radius equal to RS, draw an arc to meet the previous arc at E.
vi. Join AE and produce it to C, then ∠BAC is the required angle equal to ∠POQ.



(i) (ii)

Linear Pair axiom

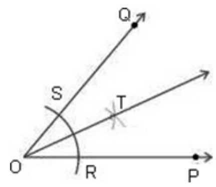
- If a ray stands on a line then the adjacent angles form a linear pair of angles.
- If two angles form a linear pair, then uncommon arms of both the angles form a straight line.

To Bisect a Given Angle

Given: Any ∠POQ

Required: To bisect ∠POQ.

Steps of Construction:

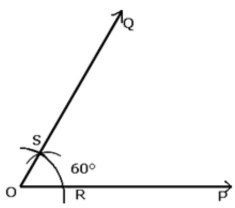


i. With O as centre and any (suitable) radius, draw an arc to meet OP at R and OQ at S.
ii. With R as centre and radius more than half of RS, draw an arc. Also, with S as centre and same radius draw another arc to meet the previous arc at T.
iii. Join OT and produce it, then OT is the required bisector of ∠POQ.

To Construct some Specific Angles

To construct an angle of 60°

Steps of Construction:



i. Draw any line OP.
ii. With O as centre and any suitable radius, draw an arc to meet OP at R.
iii. With R as centre and same radius (as in step 2), draw an arc to meet the previous arc at S.
iv. Join OS and produce it to Q, then ∠POQ = 60°.

To construct an angle of 30°

Steps of Construction

i. Construct ∠POQ = 60°.

ii. Bisect ∠POQ. Let OT be the bisector of ∠POQ, then ∠POT = 30°

To construct an angle of 120°

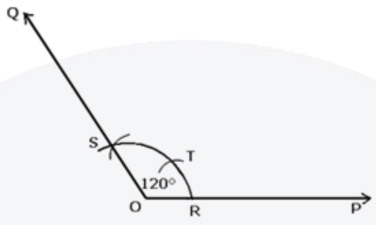
Steps of Construction:

i. Draw any line OP.

ii. With O as centre and any suitable radius, draw an arc to meet OP at R.

iii. With R as centre and same radius (as in step 2), draw an arc to meet the previous arc at T. With T as centre and same radius, draw another arc to cut the first arc at S.

iv. Join OS and produce it to Q, then ∠POQ = 120°.



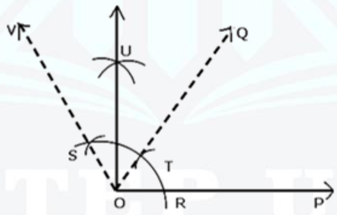
To construct an angle of 90°

Steps of Construction:

i. Construct ∠POQ = 60°

ii. Construct ∠POV = 120°.

iii. Bisect ∠QOV. Let OU be the bisector of ∠QOV, then ∠POU = 90°.



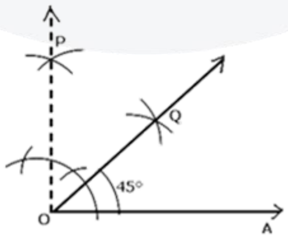
To construct an angle of 45°

Steps of Construction:

i. Construct ∠AOP = 90°.

ii. Bisect ∠AOP.

iii. Let OQ be the bisector of ∠AOP, then ∠AOQ = 45°



To Draw a Perpendicular Bisector of a Line Segment

Given: Any line segment PQ.

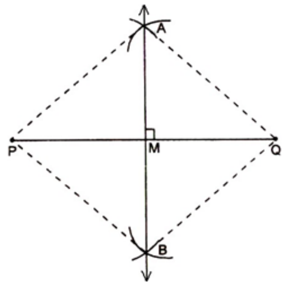
Required: To draw a perpendicular bisector of line segment PQ.

Steps of Construction:

i. With P as centre, take a length greater than half of PQ and draw arcs one on each side of PQ.

ii. With Q as centre and same radius (as in step 1), draw two arcs on each side of PQ cutting the previous arcs at A and B.

iii. Join AB to meet PQ at M, then AB bisects PQ at M, and is perpendicular to PQ, Thus, AB is the required perpendicular bisector of PQ.



Properties of a Perpendicular Bisector

- It divides AB into two equal halves or bisects it.
- It makes right angles with (or is perpendicular to) AB.
- Every point in the perpendicular bisector is equidistant from point A and B.

While working with practical geometry, you will often find the application of perpendicular bisectors; say when you are asked to draw an isosceles triangle, or when you have to determine the centre of a circle, etc. Below are the steps to construct a perpendicular bisector of a line using a compass and a ruler.

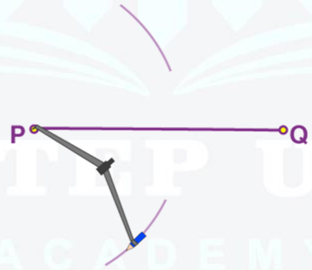
How to Construct a Perpendicular Bisector?

You will require a ruler and compasses. The steps for the construction of a perpendicular bisector of a line segment are:

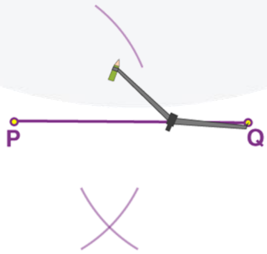
Step 1: Draw a line segment PQ.

Step 2: Adjust the compass with a length of a little more than half of the length of PQ.

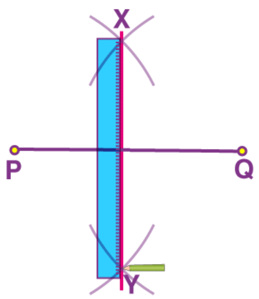
Step 3: Place the compass pointer at point P and draw arcs above and below the line.



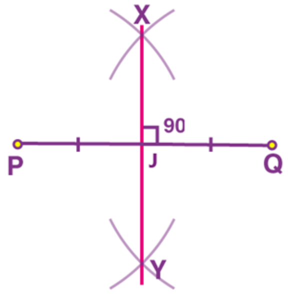
Step 4: Keeping the same length in the compass, place the compass pointer at point Q. Similarly, draw two arcs above and below the line keeping the compass pointer at Q.



Step 5: Mark the points where the opposite arcs cross as X and Y.



Step 6: Using a ruler, draw a line passing across X and Y.



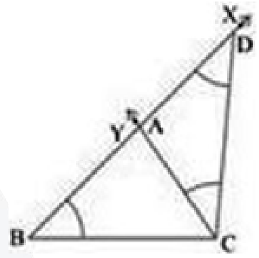
The perpendicular bisector bisects PQ at a point J, that is, the length PJ is equal to JQ. And the angle between the two lines is 90 degrees.

Construction of a Triangle, given its Base, sum of the other two sides and one Base Angle

To construct ΔABC in which base BC, ∠B and sum AC + AB of other two sides are given.

Steps of construction:

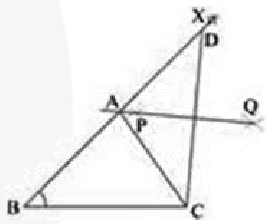
i. Draw the base BC and at the point B, make an angle, say XBC equal to the given angle.
ii. Cut a line segment BD = AC + AB from the ray BX.
iii. Join DC and make angle DCY equal to angle BDC.
iv. Let CY intersect BX at A.
v. ABC is the required triangle.



Alternate Method

Steps of construction:

i. Draw the base BC and at the point B, make an angle, say XBC equal to the given angle.
ii. Cut a line segment BD = AC + AB from the ray BX.
iii. Draw perpendicular bisector PQ of CD to intersect BD at a point A. Join AC. ABC is the required triangle.



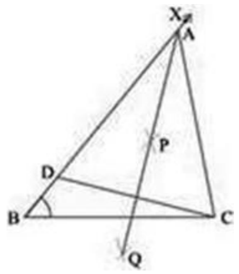
Construction of a Triangle, given its Base, difference of the other two sides and one Base

To construct ΔABC when the base BC, a base angle B and the difference of other two sides AB - AC or AC - AB are given.

Case 1: When AB > AC and AB - AC is given

Steps of construction:

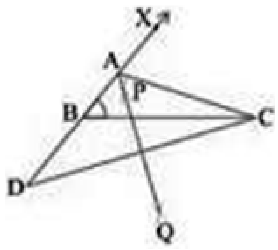
i. Draw the base BC and at point B make an angle say XBC equal to the given angle.
ii. Cut the line segment BD equal to AB - AC from ray BX.
iii. Join DC and draw the perpendicular bisector, say PQ of DC. Let it intersect BX at a point A. Join AC Then, ABC is the required triangle.



Case 1: When AB < AC and AC - AB is given Steps of Construction:

i. Draw the base BC and at point B make an angle say XBC equal to the given angle.

ii. Cut a line segment BD equal to AC - AB from the line BX extended on opposite side of line segment BC.
iii. Join DC and draw the perpendicular bisector, say PQ of DC.
iv. Let PQ intersect BX at A. Join AC.
Then, ABC is the required triangle.

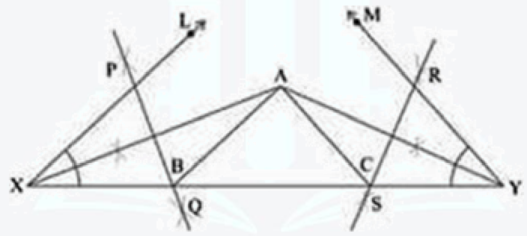


Construction of a Triangle of given Perimeter and Base Angles

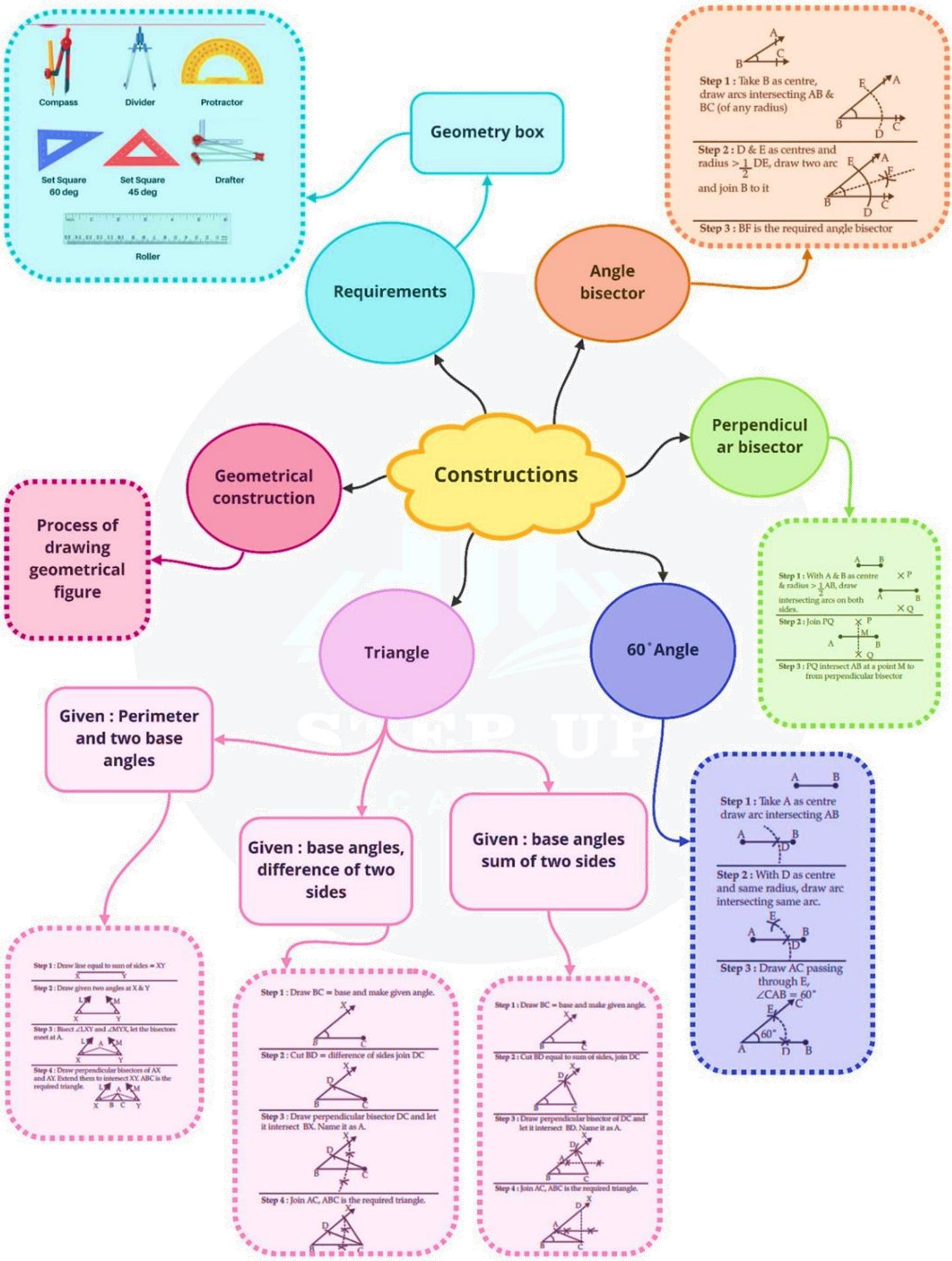
To construct a triangle ABC, when its perimeter, AB + BC + CA, and two base angles, ∠B and ∠C, are given.

Steps of Construction:

i. Draw a line segment, say XY = BC + CA + AB.
ii. Construct ∠LXY = ∠B and ∠MYX = ∠C.
iii. Draw the bisectors of ∠LXY and ∠MYX. Let these bisectors intersect at point A
iv. Draw a perpendicular bisector PQ of AX and RS of AY.
v. Let PQ intersect XY at B and RS intersect XY at C.
vi. Join AB and AC. Then, ABC is the required triangle.



Class : 9th mathematics
Chapter- 11: Constructions



Important Questions

Multiple Choice Questions

1. If a, b and c are the lengths of the three sides of a triangle, then which of the following is true?
   (a) a + b < c
   (b) a – b < c
   (c) a + b = c
2. With the help of a ruler and compasses, which of the following is not possible to construct?
   (a) 70°
   (b) 60°
   (c) 135°
3. Which of the following sets of angles can be the angles of a triangle?
   (a) 30°, 60°, 80°
   (b) 40°, 60°, 70°
   (c) 50°, 30°, 100°
4. The construction of the triangle ABC is possible if it is given that BC = 4cm, ∠C = 60° and the difference of AB and AC is
   (a) 3.5cm
   (b) 4.5cm
   (c) 3cm
   (d) 2.5cm
5. Which of the following can be the length of BC required to construct the triangle ABC such that AC = 7.4cm and AB = 5cm?
   (a) 3.5cm
   (b) 2.1cm
   (c) 4.7cm
6. If we want to construct a triangle, given its perimeter, then we need to know:
   (a) Sum of two sides of triangle
   (b) Difference between two sides of triangle
   (c) One base angles
   (d) Two base angles
7. To construct a bisector of a given angle, we need:
   (a) A ruler
   (b) A compass
   (c) A protractor
   (d) Both ruler and compass
8. Which of the following set of lengths can be the sides of a triangle?
   (a) 2cm, 4cm, 1.9cm
   (b) 1.6cm, 3.7cm, 5.3cm
   (c) 5.5cm, 6.5cm, 8.9cm
   (d) None of the above
9. Which of these angles cannot be constructed using ruler and compasses?
   (a) 120
   (b) 60
   (c) 140
   (d) 135
10. Which of the following angles can be constructed using ruler and compasses?
   (a) 35
   (b) 45
   (c) 95
   (d) 55

Very Short Questions:

1. Draw a line segment AB = 8cm. Draw 1/3 part of it. Measure the length of 1/3 part of AB.
2. Why we cannot construct a ΔABC, if ∠A = 60°, AB = 6cm and AC + BC = 5cm but construction of ΔABC is possible if ∠A = 60°, AB = 6cm and AC – BC = 5cm?
3. Construct an angle of 90° at the initial point of the given ray.
4. Draw a straight angle. Using compass bisect it. Name the angles obtained.
5. Draw any reflex angle. Bisect it using compass. Name the angles so obtained.

Short Questions:

1. Construct a triangle whose sides are in the ratio 2 : 3 : 4 and whose perimeter is 18cm.
2. Construct a? ABC with BC = 8cm? B = 45° and AB – AC = 3.1cm.
3. Construct a ΔABC such that BC = 3.2cm, ∠B = 45° and AC – AB = 2.1cm.
4. Draw a line segment QR = 5 cm. Construct perpendiculars at point Q and R to it. Name them as QX and RY respectively. Are they both parallel?

5. Construct an isosceles triangle whose two equal sides measure 6cm each and whose base is 5cm. Draw the perpendicular bisector of its base and show that it passes through the opposite vertex.

Long Questions:

1. Construct a triangle ABC in which BC = 4.7cm, AB + AC = 8.2cm and ∠C = 60°
2. Construct ΔXYZ, if its perimeter is 14cm, one side of length 5cm and ∠X = 45°
3. To construct a triangle, with perimeter 10cm and base angles 60° and 45°
4. Construct an equilateral triangle whose altitude is 6cm long
5. Construct a rhombus whose diagonals are 8 cm and 6 cm long. Measure the length of each side of the rhombus

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: a, b and c are the lengths of three sides of a triangle, then a + b > c.

   Reason: The sum of two sides of a triangle is always greater than the third side.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The side lengths 4cm, 4cm and 4cm can be sides of equilateral triangle.

   Reason: Equilateral triangle has all its three sides equal.

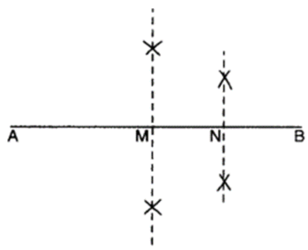
Answer Key

Multiple Choice Questions

1. (b) a – b < c
2. (a) 70°
3. (c) 50°, 30°, 100°
4. (b) 4.5cm
5. (b) 2.1cm
6. (c) One base angles
7. (d) Both ruler and compass
8. (c) 5.5cm, 6.5cm, 8.9cm
9. (c) 140
10. (b) 45

Very Short Answer:

1. **Steps of Construction:**
   - Draw a line segment AB = 8cm.
   - Draw its perpendicular bisector and let it intersect AB in M.
   - Draw the perpendicular bisector of MB and let it intersect AB in N. Thus, AN = 13 of AB = 6cm.

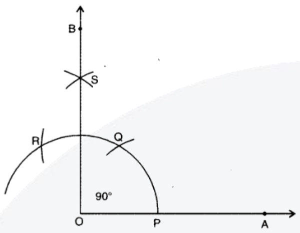


2. We know that, by triangle inequality property, construction of triangle is possible if sum of two sides of a triangle is greater than the third side. Here, AC + BC = 5cm which is less than AB (6cm) Thus, ΔABC is not possible.

   Also, by triangle inequality property, construction of triangle is possible, if difference of two sides of a triangle is less than the third side Here, AC – BC = 5cm, which is less than AB (6cm) Thus, ΔABC is possible.

3. **Steps of Construction :**

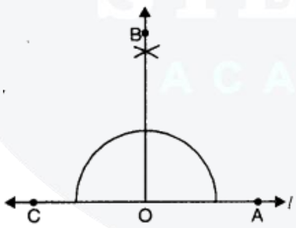
   1. Draw a ray OA.
   2. With O as centre and any convenient radius, draw an arc, cutting OA at P.



   3. With P as centre and same radius, draw an arc cutting the arc drawn in step 2 at Q.
   4. With Q as centre and the same radius as in steps 2 and 3, draw an arc, cutting the arc drawn in step 2 at R.
   5. With Q and R as centres and same radius, draw two arcs, cutting each other in S.
   6. Join OS and produce to B. Thus, ∠AOB is the required angle of 90°

4. **Steps of Construction:**

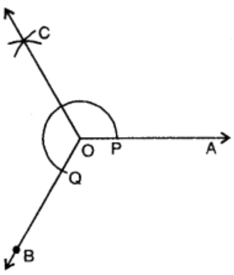
   - Draw any straight angle (say ∠AOC).
   - Bisect ∠AOC and join BO.
   - ∠AOB is the required bisector of straight angle AOC.



5. **Steps of Construction:**

   a. Let ∠AOB be any reflex angle.

   b. With O as Centre and any convenient radius, draw an arc cutting OA in P and OB in Q.

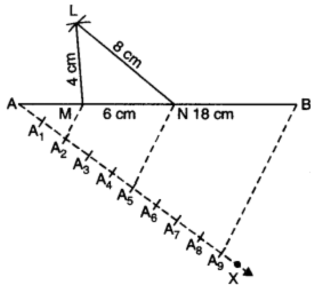


   c. With P and Q as centers, draw two arcs of radius little more than half of it and let they intersect each other in C. Join OC. Thus, OC is the required bisector. Angles so obtained are ∠AOC and ∠COB.

Short Answer:

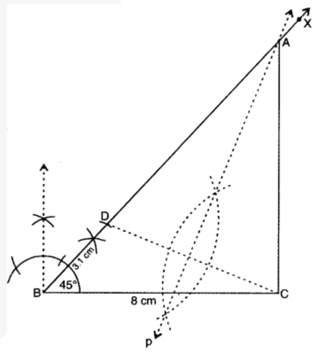
1. **Steps of Construction:**

   - Draw a line segment AB = 18cm.
   - At A, construct an acute angle ∠BAX (< 90°).
   - Mark 9 points on AX, such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6$
   - $= A_6A_7 = A_7A_8 = A_8A_9$.
   - Join A_9B.
   - From A_2 and A5, draw $A_2M \parallel A_5N \parallel A_9B$, intersecting AB in M and N respectively.
   - With M as Centre and radius AM, draw an arc.
   - With N as Centre and radius NB, draw another arc intersecting the previous arc at L.
   - Join LM and LN. Thus, ΔLMN is the required triangle.



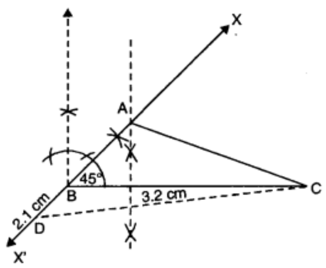
2. **Steps of Construction:**

   - Draw any line segment BC = 8cm.
   - At B, construct an angle ∠CBX = 45°.
   - From BX, cut off BD = 3.1cm.
   - Join DC.
   - Draw the perpendicular bisector ‘p’ of DC and let it intersect BX in A.
   - Join AC. Thus, ΔABC is the required triangle.

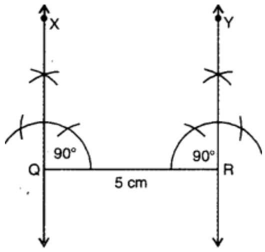


3. **Steps of Construction:**

   - Draw a line segment BC = 3.2cm.
   - At B, construct an angle ∠CBX = 45° and produce it to point X’.
   - Cut-off BD = 2.1cm and join CD.
   - Draw the perpendicular bisector of CD and let it intersect X’BX in A.
   - Join AC. Thus, ΔABC is the required triangle.

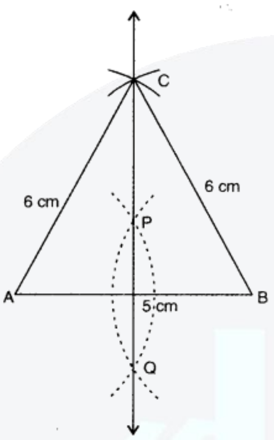


4. **Steps of Construction:**



- Draw a line segment QR = 5cm.
- With Q as Centre, construct an angle of 90° and let this line through Q is QX.
- With R as Centre, construct an angle of 90° and let this line through R is RY. Yes, the perpendicular lines QX and RY are parallel.

5. **Steps of Construction:**

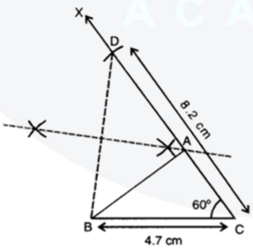


- Draw a line segment AB = 5cm.
- With A and B as centers, draw two arcs of radius 6 cm and let they intersect each other in C.
- Join AC and BC to get ΔABC.
- With A and B as canters, draw two arcs of radius little more than half of AB. Let they intersect each other in P and Q. Join PQ and produce, to pass through C.

Long Answer:

1. **Given**: In ΔABC, BC = 4.7cm, AB + AC = 8.2cm and ∠C = 60°.

Required: To construct ΔABC.



Steps of Construction:

- Draw BC = 4.7cm.
- Draw
- From ray CX, cut off CD = 8.2cm.
- Join BD.
- Draw the perpendicular bisector of BD meeting CD at A.
- Join AB to obtain the required triangle ABC.

Justification:

∵ A lies on the perpendicular bisector of BD, therefore, AB = AD

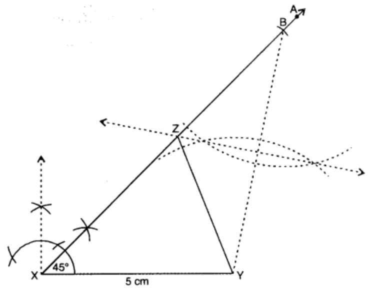
Now, CD = 8.2cm

⇒ AC + AD = 8.2cm

⇒ AC + AB = 8.2cm

2. Here, perimeter of ΔXYZ = 14cm and one side XY = 5cm

∴ YZ + XZ = 14 – 5 = 9cm and ∠X = 45°.



Steps of Construction:

- Draw a line segment XY = 5cm.
- Construct an ∠YXA = 45° with the help of compass and ruler.
- From ray XA, cut off XB = 9cm.
- Join BY.
- Draw perpendicular bisector of BY and let it intersect XB in Z.
- Join ZY. Thus, ΔXYZ is the required triangle.

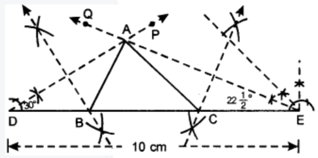
3. **Given**: In ΔABC,

AB + BC + CA = 10 cm, ∠B = 60° and ∠C = 45°.

Required: To construct ΔABC.

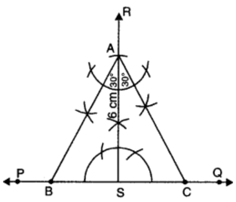
Steps of Construction:

- Draw DE = 10cm.
- At D, construct ∠EDP= 5 of 60°= 30° and at E, construct DEQ = 1 of 45o = 22°



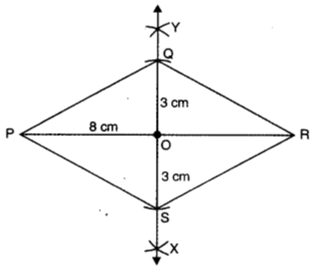
- Let DP and EQ meet at A.
- Draw perpendicular bisector of AD to meet DE at B.
- Draw perpendicular bisector of AE to meet DE at C.
- Join AB and AC. Thus, ABC is the required triangle.

4. **Steps of Construction:**



- Draw a line PQ and take any point S on it.
- Construct the perpendicular SR on PQ.
- From SR, cut a line segment SA = 6cm.
- At the initial point A of the line segment AS, construct ∠SAB = 30° and ∠SAC = 30°.
- The arms AB and AC of the angles ∠SAB and ∠SAC meet PQ in B and C respectively. Then, ΔABC is the required equilateral triangle with altitude of length 6cm.

5. Steps of Construction:



- Draw a line segment PR = 8cm.
- Draw the perpendicular bisector XY of the line segment PR.
  Let O be the point of intersection of PR and XY, so that O is the 8 cm mid-point of PR.
- From OX, cut a line segment OS = 3cm and from OY, cut a line segment OQ = 3cm.
- Join PS, SR, RQ and QP, then PQRS is the required rhombus.
- Measure the length of segments PQ, QR, RS and SP, each is found to be 5cm long.

Assertion and Reason Answers-

1. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Heron's Formula 12

Heron's Formula

1. The region enclosed within a simple closed figure is called its **area**.
2. **Area of a triangle** = $\frac{1}{2}$ × base × height
3. **Area of an equilateral triangle** = $\frac{\sqrt{3}}{4}$ a^2 sq units, where 'a' is the side length of an equilateral triangle.
4. **Semi-perimeter** is half of the perimeter.
5. If a, b and c denote the lengths of the sides of a triangle, then the area of the triangle is calculated by using **Heron's formula**, as given below:
6. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, s = semi – perimeter = $\frac{a+b+c}{2}$
7. For every triangle, the values of (s – a), (s – b), and (s – c) are positive.
8. Area of a quadrilateral can be calculated by dividing the quadrilateral into two triangles and using Heron's formula for calculating area of each triangle.

Triangle

The plane closed figure, with three sides and three angles is called as a triangle.

Types of triangles:

Based on sides a) Equilateral b) Isosceles c) Scalene

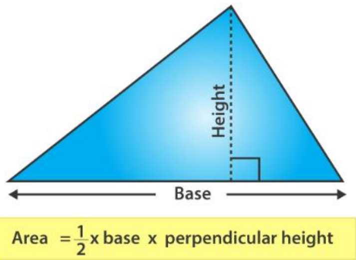
Based on angles a) Acute angled triangle b) Right-angled triangle c) Obtuse angled triangle

Area of a triangle

Area = (1/2) × base × height

In case of equilateral and isosceles triangles, if the lengths of the sides of triangles are given then, we use Pythagoras theorem in order to find the height of a triangle.

Area of a triangle is the region enclosed by it, in a two-dimensional plane. As we know, a triangle is a closed shape that has three sides and three vertices. Thus, the area of a triangle is the total space occupied within the three sides of a triangle. The general formula to find the area of the triangle is given by half of the product of its base and height.



In general, the term "area" is defined as the region occupied inside the boundary of a flat object or figure. The measurement is done in square units with the standard unit being square meters (m2). For the computation of area, there are pre-defined formulas for squares, rectangles, circle, triangles, etc. In this article, we will learn the area of triangle formulas for different types of triangles, along with some example problems.

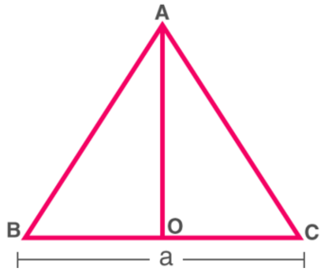
Example: What is the area of a triangle with base b = 3 cm and height h = 4 cm?

Using the formula,

Area of a Triangle, A = 1/2 × b × h = 1/2 × 4 cm × 3 cm = 2 cm × 3 cm = 6 cm^2

Apart from the above formula, we have Heron's formula to calculate the triangle's area, when we know the length of its three sides. Also, trigonometric functions are used to find the area when we know two sides and the angle formed between them in a triangle. We will calculate the area for all the conditions given here.

Area of an equilateral triangle



Consider an equilateral ΔABC, with each side as a unit. Let AO be the perpendicular bisector of BC. In order to derive the formula for the area of an equilateral triangle, we need to find height AO.

Using Pythagoras theorem,

$AC^2 = OA^2 + OC^2$

$OA^2 = AC^2 - OC^2$

Substitute AC = a, OC = a/2 in the above equation.

$OA^2 = a^2 - a^2/4$

$OA = \sqrt{3}\,a/2$

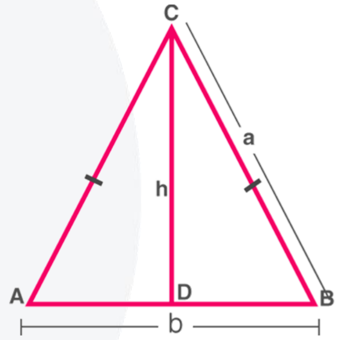
We know that the area of the triangle is:

A = (1/2) × base × height

$A = (1/2) \times a \times (\sqrt{3}\,a/2)$

∴ Area of Equilateral triangle = $\sqrt{3}\,a^2/4$

Area of an isosceles triangle



Consider an isosceles ΔABC with equal sides as a units and base as b units.

Isosceles triangle ABC

The height of the triangle can be found by Pythagoras' Theorem:

$CD^2 = AC^2 - AD^2$

$\Rightarrow h^2 = a^2 - (b^2/4) = (4a^2 - b^2)/4$

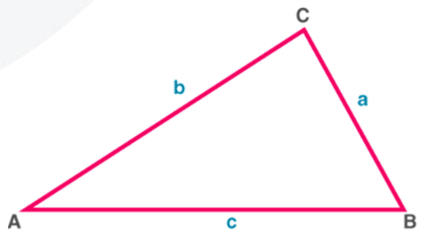
$\Rightarrow h = \frac{1}{2}\sqrt{(4a^2 - b^2)}$

Area of triangle is A = (1/2) bh

$\therefore A = \left(\frac{1}{2}\right) \times b \times \left(\frac{1}{2}\right)\sqrt{(4a^2 - b^2)}$

$\therefore A = \left(\frac{1}{4}\right) \times b \times \sqrt{(4a^2 - b^2)}$

Area of a triangle – By Heron's formula



Triangle ABC

Area of a ΔABC, given sides a, b, c by Heron's formula (also known as Hero's Formula) is:

Find semi perimeter (s) = (a + b + c)/2

$\text{Area} = \sqrt{[s(s-a)(s-b)(s-c)]}$

This formula is helpful to find the area of a scalene triangle, given the lengths of all its sides.

Heron's Formula

Heron's formula is used to find the area of a triangle when we know the length of all its sides. It is also termed as Hero's Formula. We can Heron's formula to find different types of triangles, such as scalene, isosceles and equilateral triangles. We don't have to need to know the angle measurement of a triangle to calculate its area, by using Heron's formula.

Heron's formula is a formula to calculate the area of triangles, given the three sides of the triangle. This formula is also used to find the area of the quadrilateral, by dividing the quadrilateral into two triangles, along its diagonal.

If a, b and c are the three sides of a triangle, respectively, then Heron's formula is given by:

Area of triangle using three sides = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi perimeter, s = Perimeter of triangle/2 = (a + b + c)/2

History of Heron's Formula

Hero of Alexandria was a great mathematician who derived the formula for the calculation of the area of a triangle using the length of all three sides. He also extended this idea to find the area of quadrilateral and also higher-order polygons. This formula has its huge applications in trigonometry such as proving the law of cosines or the law of cotangents, etc.

Proof of Heron's Formula

There are two methods by which we can derive Heron's formula.

- First, by using trigonometric identities and cosine rule.
- Secondly, solving algebraic expressions using the Pythagoras theorem.

Let us see one by one both the proofs or derivation.

Using Cosine Rule

Let us prove the result using the law of cosines:

Let a, b, c be the sides of the triangle and α, β, γ are opposite angles to the sides.

We know that the law of cosines is

$$\text{Cos } \gamma = \frac{a^2+b^2-c^2}{2ab}$$

Again, using trig identity, we have

$$\text{Sin } \gamma = \sqrt{1-\cos^2\gamma}$$

$$= \frac{\sqrt{4a^2b^2-(a^2+b^2-c^2)^2}}{2ab}$$

Here, Base of triangle = a

Altitude = b sin γ

Now,

$$A = \frac{1}{2}\text{(base) (altitude)}$$

$$= \frac{1}{2}ab \sin\gamma$$

$$= \frac{1}{4}\sqrt{4a^2b^2-(a^2+b^2-c^2)^2} = \frac{1}{4}\sqrt{(2ab-(a^2+b^2-c^2))(2ab+a^2+b^2-c^2)^2}$$

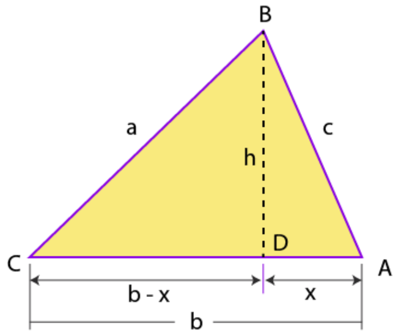
$$= \frac{1}{4}\sqrt{(c^2-(a-b)^2)((a+b)^2-c^2)} = \sqrt{\frac{(c-(a-b))(c+(a-b))((a+b)-c)((a+b)+c)}{16}}$$

$$= \sqrt{\frac{(b+c-a)}{2}\frac{(a+c-b)}{2}\frac{(a+b-c)}{2}\frac{(a+b+c)}{2}}$$

$$= \sqrt{\frac{(a+b+c)}{2}\frac{(b+c-a)}{2}\frac{(a+c-b)}{2}\frac{(a+b-c)}{2}}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}.$$

Using Pythagoras Theorem



Area of a Triangle with 3 Sides

Area of ΔABC is given by

A = 1/2 bh ____ (i)

Draw a perpendicular BD on AC

Consider a ΔADB

$x^2 + h^2 = c^2$

$x^2 = c^2 - h^2$—(ii)

$\Rightarrow x = \sqrt{(c^2 - h^2)}$ -------—(iii)

Consider a ΔCDB,

$(b - x)^2 + h^2 = a^2$

$(b - x)^2 = a^2 - h2$

$b^2 - 2bx + x^2 = a^2 - h^2$

Substituting the value of x and x^2 from equation (ii) and (iii), we get

$b^2 - 2b\sqrt{(c^2 - h^2)} + c^2 - h^2 = a^2 - h^2$

$b^2 + c^2 - a^2 = 2b\sqrt{(c^2 - h^2)}$

Squaring on both sides, we get;

$(b^2 + c^2 - a^2)^2 = 4b^2(c^2 - h^2)$

$$\frac{(b^2+c^2-a^2)^2}{4b^2} = c^2 - h^2$$

$$h^2 = c^2 - \frac{(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{4b^2c^2 - (b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{(2bc)^2 - (b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{\left[2bc + (b^2+c^2-a^2)\right]\left[2bc - (b^2+c^2-a^2)\right]}{4b^2}$$

$$h^2 = \frac{\left[(b^2+2bc+c^2) - a^2\right]\left[a^2 - (b^2+2bc+c^2)\right]}{4b^2}$$

$$h^2 = \frac{\left[(b+c)^2 - a^2\right]\cdot\left[a^2 - (b-c)^2\right]}{4b^2}$$

$$h^2 = \frac{\left[(b+c)+a\right]\left[(b+c)-a\right]\cdot\left[a+(b-c)\right]\left[a-(b-c)\right]}{4b^2}$$

$$h^2 = \frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{4b^2}$$

The perimeter of a ΔABC is

P = a + b + c

$$\Rightarrow h^2 = \frac{P(P-2a)(P-2b)(P-2c)}{4b^2}$$

$$\Rightarrow h = \sqrt{P(P-2a)(P-2b)(P-2c)}2b$$

Substituting the value of h in equation (i), we get;

$$A = \frac{1}{2}b\frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

$$A = \frac{1}{4}\sqrt{P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{1}{16}P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{P}{2}\left(\frac{P-2a}{2}\right)\left(\frac{P-2b}{2}\right)\left(\frac{P-2c}{2}\right)}$$

Semi perimeter(s) = $\frac{\text{perimeter}}{2} = \frac{P}{2}$

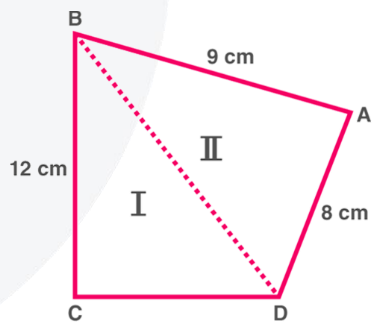
$\Rightarrow A = \sqrt{s(s-a)(s-b)(s-c)}$

Note: Heron's formula is applicable to all types of triangles and the formula can also be derived using the law of cosines and the law of Cotangents.

Area of any polygon – By Heron's formula

For a quadrilateral, when one of its diagonal value and the sides are given, the area can be calculated by splitting the given quadrilateral into two triangles and use the Heron's formula.

Example: A park, in the shape of a quadrilateral ABCD, has $\angle C = 90^\circ$, AB = 9 cm, BC = 12 cm, CD = 5 cm and AD = 8 cm. How much area does it occupy?



⇒ We draw the figure according to the information given.

The figure can be split into 2 triangles ΔBCD and ΔABD

From ΔBCD, we can find BD (Using Pythagoras' Theorem)

$BD^2 = 12^2 + 5^2 = 169$

BD = 13cm

Semi-perimeter for ΔBCD $S_1 = (12 + 5 + 13)/2 = 15$

Semi-perimeter ΔABD $S_2 = (9 + 8 + 13)/2 = 15$

Using Heron's formula A_1 and A_2 will be:

$A_1 = \sqrt{[15(15-12)(15-5)(15-13)]}$

$A_1 = \sqrt{(15\times3\times10\times2)}$

A1 = $\sqrt{900}$ = 30 cm^2

Similarly,

A_2 will be 35.49 cm^2.

The area of the quadrilateral ABCD = $A_1 + A_2$ = 65.49 cm^2

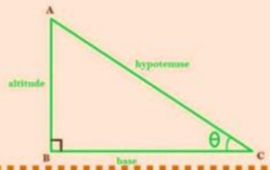
Class : 9th mathematics
Chapter- 12: Heron's Formula

Heron's Formula

Area

Right - angled triangle

$\frac{1}{2} \times \text{Base} \times \text{Altitude}$



Equilateral triangle

$\frac{\sqrt{3}}{4} a^2$; a = side of triangle

Applications

Find area of triangle of sides 122cm, 22cm, 120cm

By Heron's formula

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

where, $s = \frac{a+b+c}{2}$

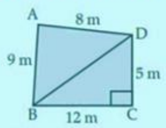
Here, $s = \frac{122+22+120}{2} = 132$ cm

$\therefore$ Area $= \sqrt{132(132\text{-}122)(132\text{-}22)(132\text{-}120)}$ cm^2

$= (\sqrt{132(10)(110)(12)})$cm^2

$= 1320$ cm^2

Area of quadrilateral ABCD with given dimensions :-



Area of ABCD = Area of ΔABD + area of ΔBCD

Here, area of ΔBCD $= \frac{1}{2} \times$ BC $\times$ CD

$= \frac{1}{2} \times 12 \times 5 = 30$ m^2

Here BD, $= \sqrt{BC^2 + DC^2}$

BD $= \sqrt{12^2 + 5^2} = 13$ m

Area of ABD $= \sqrt{s(s\text{-}a)(s\text{-}b)(s\text{-}c)}$

where $s = \frac{a+b+c}{2} = \frac{(9+8+13)}{2}$ m $= 15$ m

Area $= \sqrt{15(15\text{-}9)(15\text{-}8)(15\text{-}13)}$ m^2

ABD $= \sqrt{15(6)(7)(2)}$ m^2

$= 35.496$ m^2

$\therefore$ Area of ABCD $= (30 + 35.496)$ m^2

$= 65.496$ m^2

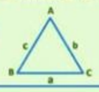
Heron's Formula

Heron's Formula

Area of a triangle with its sides as a, b and c is calculated by using Heron's Formula stated as

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

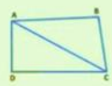
Where, S = semi - perimeter $= \frac{a+b+c}{2}$.



Applications of Heron's Formula

Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and then using the Heron's Formula to find the area of the two triangles thus formed.

Then we add the area of two triangles to get the area of the quadrilateral.



Important Questions

Multiple Choice Questions

1. An isosceles right triangle has area $8cm^2$. The length of its hypotenuse is
   (a) $\sqrt{32}$ cm
   (b) $\sqrt{16}$ cm
   (c) $\sqrt{48}$ cm
   (d) $\sqrt{24}$ cm
2. The perimeter of an equilateral triangle is 60m. The area is
   (a) $10\sqrt{3}\ m^2$
   (b) $15\sqrt{3}\ m^2$
   (c) $20\sqrt{3}\ m^2$
   (d) $100\sqrt{3}\ m^2$
3. The sides of a triangle are 56cm, 60cm and 52cm long. Then the area of the triangle is
   (a) $1322cm^2$
   (b) $1311cm^2$
   (c) $1344cm^2$
   (d) $1392cm^2$
4. The area of an equilateral triangle with side $2\sqrt{3}$ cm is
   (a) $5.196cm^2$
   (b) $0.866cm^2$
   (c) $3.496cm^2$
   (d) $1.732cm^2$
5. The length of each side of an equilateral triangle having an area of $9\sqrt{3}\ cm^2$ is
   (a) 8cm
   (b) 36cm
   (c) 4cm
   (d) 6cm
6. If the area of an equilateral triangle is $16\sqrt{3}\ cm^2$, then the perimeter of the triangle is
   (a) 48cm
   (b) 24cm
   (c) 12cm
   (d) 36cm
7. The sides of a triangle are 35cm, 54cm and 61cm. The length of its longest altitude is
   (a) $16\sqrt{5}$ cm
   (b) $10\sqrt{5}$ cm
   (c) $24\sqrt{5}$ cm
   (d) 28cm
8. The area of an isosceles triangle having base 2cm and the length of one of the equal sides 4 cm is
   (a) $15\sqrt{}cm^2$
   (b) $\sqrt{15}/2\ cm^2$
   (c) $2\sqrt{15}\ \ 5cm^2$
   (d) $4\sqrt{15}\ cm^2$
9. The edges of a triangular board are 6 cm, 8cm and 10cm. The cost of painting it at the rate of 9 paise per cm^2 is
   (a) Rs 2.00
   (b) Rs 2.16
   (c) Rs 2.48
   (d) Rs 3.00
10. The base of a right triangle is 48cm and its hypotenuse is 50cm. The area of the triangle is
    (a) $168cm^2$
    (b) $252cm^2$
    (c) $336cm^2$
    (d) $504cm^2$

Very Short Questions:

1. Find the area of an equilateral triangle having side 6cm.
2. If the perimeter of an equilateral triangle is 90m, then find its area.
3. If every side of a triangle is doubled, then find the percent increase in area of triangle so formed.
4. If the length of a median of an equilateral triangle is x cm, then find its area.

Short Questions:

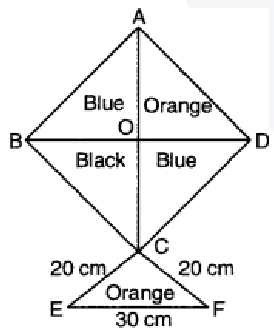
1. Find the area of a triangle whose sides are 11m, 60m and 61m.
2. Suman has a piece of land, which is in the shape

of a rhombus. She wants her two sons to work on the land and produce different crops. She divides the land in two equal parts by drawing a diagonal. If its perimeter is 400 m and one of the diagonals is of length 120 m, how much area each of them will get for his crops?

3. The perimeter of a triangular field is 144m and its sides are in the ratio 3 : 4 : 5. Find the length of the perpendicular from the opposite vertex to the side whose length is 60m.
4. Find the area of the triangle whose perimeter is 180 cm and two of its sides are of lengths 80 cm and 18 cm. Also, calculate the altitude of the triangle corresponding to the shortest side.

Long Questions:

1. Calculate the area of the shaded region.
2. The sides of a triangular park are 8m, 10m and 6m respectively. A small circular area of diameter 2m is to be left out and the remaining area is to be used for growing roses. How much area is used for growing roses? (Use n = 3.14)
3. OPQR is a rhombus, whose three vertices P, Q and R lie on the circle with Centre 0. If the radius of the circle is 12cm, find the area of the rhombus.
4. How much paper of each shade is needed to make a kite given in the figure, in which ABCD is a square with diagonal 60cm?



Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: the area of a triangle is 6 cm^2 whose sides are 3 cm, 4 cm and 5 cm respectively.

   Reason: area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: the area of an equilateral triangle having each side 4 cm is $4\sqrt{3}$ cm^2

   Reason: Area of an equilateral triangle = $\left(\frac{\sqrt{3}}{4}\right)\times a^2$

Answer Key

Multiple Choice Questions

1. (b) $\sqrt{32}$ cm
2. (d) $100\sqrt{3}$ m^2
3. (c) 1344 cm^2
4. (a) 5.196 cm^2
5. (d) 6 cm
6. (b) 24 cm
7. (c) $24\sqrt{5}$ cm
8. (a) $15\sqrt{}$cm^2
9. (b) Rs 2.16
10. (c) 336 cm^2

Very Short Answer:

1. Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$ =

$\frac{\sqrt{3}}{4} \times 6 \times 6 = 9\sqrt{3}$ cm^2

2. Side of an equilateral triangle = $\frac{\text{Perimeter}}{3} = \frac{90}{3}$

= 30 m

∴ Its area = $\frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (30)^2$

$= \frac{\sqrt{3}}{4} \times 30 \times 30 = 225\sqrt{3}$ m^2

3. Let the sides of the given triangle be, a units, b units and c units.

∴ Its area = $\sqrt{s(s-a)(s-b)(s-c)}$ sq. units

Now, new sides of the triangle are 2a units, 2b units and 2c units.

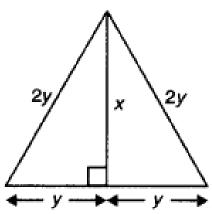
Thus, Its area = $\sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)}$ sq. units

Total increase in area = $3\sqrt{s(s-a)(s-b)(s-c)}$ sq. units

Hence, percent increase = 300%

4. Let each equal sides of given equilateral triangle be $2y^2$. We know that median is also perpendicular bisector.



∴ $y^2 + x^2 = 4y^2$

$\Rightarrow x^2 = 3y^2$

$\Rightarrow x = \sqrt{3}\,y$

or

$\Rightarrow y = \frac{x}{\sqrt{3}}$

Now, area of given triangle = $\frac{1}{2} \times 2y \times X = y \times x = \frac{x}{\sqrt{3}} \times x = \frac{x^2}{\sqrt{3}}$

Short Answer:

1. Let a = 11m, b = 60m and c = 61m.

∴ $s = \frac{a+b+c}{2} = \frac{11+60+61}{2} = \frac{132}{2} = 66$ m

Now, s – a = 66 – 11 = 55 m

s – b = 66 – 60 = 6 m

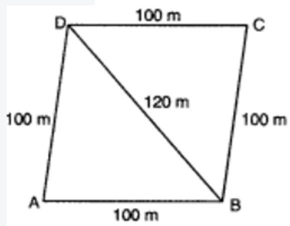
s – c = 66 – 61 = 5 m

∴ Area of given triangle

$= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{66(55)(6)(5)}$

$= \sqrt{108900} = 330$ sq. m.

2. Here, perimeter of the rhombus is 400m.



∴ Side of the rhombus = 400/4 = 100m

Let diagonal BD = 120m and this diagonal divides the rhombus ABCD into two equal parts.

Now, $s = \frac{100+120+100}{2} = \frac{320}{2} = 160$

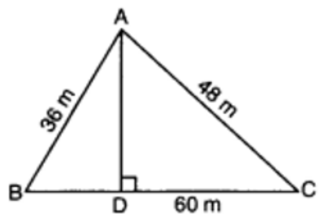
∴ Area of ΔABD

$= \sqrt{160(160-100)(160-100)(160-120)}$

$= \sqrt{160 \times 60 \times 60 \times 40} = 80 \times 60$ m^2

Hence, area of land allotted to two sons for their crops is 4800m^2 each.

3. Let the sides of the triangle be 3x, 4x and 5x



∴ The perimeter of the triangular field = 144m

$\Rightarrow 3x + 4x + 5x = 144$

$\Rightarrow 12x = 144$

$\Rightarrow x = \frac{144}{12} = 14$

∴ Sides of the triangle are 3×12m, 4×12m, 5×12m i.e., 36 m, 48 m, 60 m

∴ $s = \frac{a+b+c}{2} = \frac{36+48+60}{2} = \frac{144}{2} = 72$ m

∴ Area of the ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{72(72-36)(72-48)(72-60)}$

$= \sqrt{72(36)(24)(12)} = \sqrt{746496} = 864\ m^2$

∴ Also, ar (ΔABC) = $\frac{1}{2} \times AD \times BC = \frac{1}{2} \times AD \times 60$

= 30 × AD

∴ 30 × AD = 864

$AD = \frac{864}{30} = 28.8$ m

4. Perimeter of given triangle = 180cm

Two sides are 18cm and 80cm

∴ Third side = 180 – 18 – 80 = 82cm

$s = \frac{180}{2} = 90$ cm

Area of triangle

$= \sqrt{90(90-18)(90-80)(90-82)}$

$= \sqrt{90 \times 72 \times 10 \times 8} = \sqrt{518400} = 720\ cm^2$

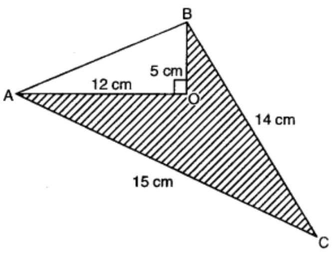
Also, $\frac{1}{2} \times 18 \times h = 720$

$h = \frac{720}{9} = 80$ cm

Hence, area of triangle is 720cm² and altitude of the triangle corresponding to the shortest side is 80cm.

Long Answer:

1. Area of ΔAOB = $\frac{1}{2} \times OA \times OB$



$= \frac{1}{2} \times 12 \times 5$

= 30 cm²

Also, $AB^2 = OA^2 + OB^2$

$= 12^2 + 5^2$

= 144 + 25 = 169

$\Rightarrow AB = \sqrt{169} = 13$ cm

Now, in ΔABC, we have

a = BC = 14 cm, b = CA = 15 cm, c = AB = 13 cm

$s = \frac{a+b+c}{2} = \frac{14+15+13}{2} = \frac{42}{2} = 21$ cm

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-14)(21-15)(21-13)}$

$= \sqrt{21 \times 7 \times 6 \times 8} = \sqrt{3 \times 7 \times 7 \times 2 \times 3 \times 2 \times 2 \times 2}$

2. The sides of the triangular park are 8m, 10m and 6m.

∴ $s = \frac{a+b+c}{2} = \frac{14+15+13}{2} = \frac{42}{2} = 21$ m

Area of the park = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-8)(12-10)(12-6)}$

$= \sqrt{21 \times 4 \times 2 \times 6} = \sqrt{2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3}$

= 2 × 2 × 2 × 3 = 24 m²

Radius of the circle = $\frac{2}{2} = 1$ m

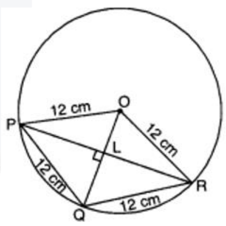
Area of the circle = πr^2 = 3.14 × 1 × 1 = 3.14 m²

∴ Area to be used for growing roses = Area of the park - area of the circle = 24 – 3.14 = 20.86 m²

3. Since diagonals bisect each other at 90°.

∴ In right ΔQLR,

$(LR)^2 + (LQ)^2 = (QR)^2$



$\Rightarrow \left(\frac{PR}{2}\right)^2 + \left(\frac{OQ}{2}\right)^2 = (QR)^2$

$\Rightarrow \left(\frac{PR}{2}\right)^2 = (12)^2 - \left(\frac{12}{2}\right)^2$

[∵ OQ = r = 12 cm]

$\Rightarrow \frac{PR^4}{4} = 144 - 36$

$\Rightarrow PR^2 = 4 \times 108 = 432$

$PR = \sqrt{144 \times 3} = 12\sqrt{3}$ cm.

Area of rhombus OPQR = $\frac{1}{2}$ × product of diagonals

$= \frac{1}{2} \times OQ \times PR = \frac{1}{2} \times 12 \times 12\sqrt{3}$

$= 72\sqrt{3}$ cm²

4. Since diagonals of a square are of equal length and bisect each other at right angles, therefore,

Area of ΔAOD = $\frac{1}{2} \times 30 \times 30 = 450$ cm²

Area of ΔAOD = Area of ΔDOC = Area of ΔBOC = Area of ΔAOB = 450 cm²

[$\because$ ΔAOD = ΔAOB ≅ ΔBOC ≅ ΔCOD,
$\because$ they have equal area]

Now, area of ACEF (by Heron's formula)

Here a = 20cm, b = 20cm and c = 30cm

$\Rightarrow \ s = \frac{20+20+30}{2} = \frac{70}{2}$ = 35 cm

Area of ΔCEF = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{35(35-20)(35-20)(35-30)}$

$= \sqrt{35(15)(15)(5)} = 75\sqrt{7}$ $cm^2 \cong 198.4\ cm^2$

Now, area of orange shaded paper in kite

= Area of ΔAOD + Area of ΔCEF

= 450 cm^2 + 198.4 cm^2

= 648.4 cm^2

Area of blue shaded paper in kite

= Area of ΔAOB + Area of ΔCOD

= 450 cm^2 + 450 cm^2 = 900 cm^2

Area of black shaded paper in kite = Area of ΔBOC

= 450 cm^2.

Assertion and Reason Answers-

1. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Surface Areas and Volumes 13

Surface Areas and Volumes

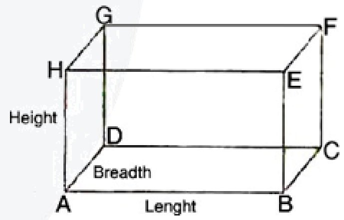
1. A cuboid is a solid bounded by six rectangular plane regions. It has length, width and height.
2. A cuboid whose all edges are equal is called a cube.
3. A cylinder is a closed solid that has two parallel (usually circular) bases connected by a curved surface.
4. A cone is a solid that has a circular base and a single vertex.
5. A sphere is a perfectly round geometrical object in three-dimensional space, such as the shape of a round ball.
6. A hemisphere is half of a sphere.
7. Surface area of a solid is the sum of the areas of all its faces.
8. The total surface area of any object will be greater than its lateral surface area.
9. In case of a room, lateral surface area means the area of the four walls of the room, whereas total surface area means the area of four walls plus the area of the floor and the ceiling.
10. Volume is the space occupied by an object.
11. The unit of measurement of both volume and capacity is cubic unit such as cubic feet, cubic cm. cubic m etc.
12. If l, b, h denote respectively the length, breadth and height of a **cuboid**, then:

    Lateral surface area or Area of four walls = $2(\ell + b)h$

    Total surface area = $2(\ell b + bh + h\ell)$

    Volume = $\ell \times b \times h$

    Diagonal of a cuboid = $\sqrt{\ell^2 + b^2 + h^2}$



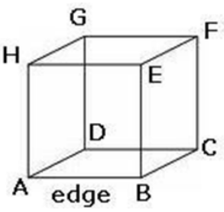
13. If the length of each edge of a **cube** is 'a' units, then:

    Lateral surface area = $4 \times (\text{edge})^2$

    Total surface area = $6 \times (\text{edge})^2$

    Volume = $(\text{edge})^3$

    Diagonal of a cube = $\sqrt{3} \times \text{edge}$

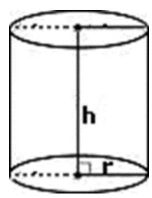


14. If r and h respectively denote the radius of the base and the height of a **right circular cylinder**, then: Area of each end or Base area = πr^2

    Area of curved surface or lateral surface area = perimeter of the base x height = $2\pi rh$

    Total surface area (including both ends) = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

    Volume = Area of the base × height = $\pi r^2 h$

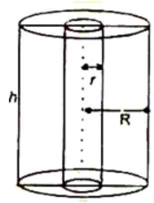


15. If R and r respectively denote the external and internal radii of a **right circular hollow cylinder** and h denotes its height, then:

    Area of each circular base = $\pi R^2 - \pi r^2$

    Area of curved surface = $2\pi(R + r)h$

    Total surface area = (External surface) + (Internal surface)

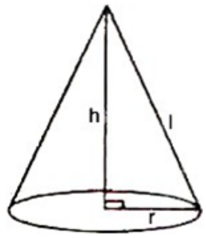


$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$

Volume of = (External volume) – (Internal volume)

$= (\pi R^2h - \pi r^2h) = \pi h(R^2 - r^2)$

16. If r, h and l respectively denote the radius, height and slant height of a **right circular cone**, then: Slant height $(\ell) = \sqrt{h^2 + r^2}$

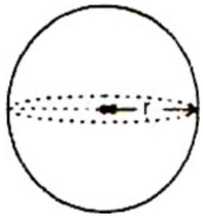


Area of curved surface $\pi r\ell = \pi r\sqrt{h^2 + r^2}$

Total surface area = Area of curved surface + Area of base $= \pi r\ell + \pi r^2 = \pi r(\ell + r)$

Volume $= \frac{1}{3}\pi r^2h$

17. If r is the radius of a **sphere**, then: Surface area $= 4\pi r^2$



Volume $= \frac{4}{3}\pi r^3$

18. If r is the radius of a **hemisphere**, then:

Area of curved surface $= 2\pi r^2$

Total surface Area = Area of curved surface + Area of base

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

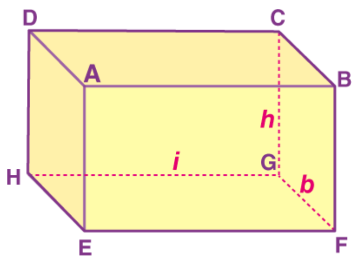
Volume $= \frac{2}{3}\pi r^3$

19. Volume of water flown in the tank in one hour = (area of cross section of the aperture) × (speed in meters per hour)
20. When an object of certain volume is recast into a cylinder, the volume of the cylinder formed will always be equal to the volume of the original object.
21. The solids having the same curved surface do not necessarily occupy the same volume.
22. When an object is dropped into a liquid, the volume of the displaced liquid is equal to the volume of the object that is dipped.
23. Of all the solids having a given volume, the sphere is the one with the smallest surface area. Of all solids having a given surface area, the sphere is the one having the greatest volume.

Cuboid

A cuboid is a three-dimensional Shape. The cuboid is made from six rectangular faces, which are placed at right angles. The total surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

Total Surface Area of a Cuboid



Consider a cuboid whose length is "l" cm, breadth is b cm and height h cm.

Area of face ABCD = Area of Face EFGH = $(l \times b)$ cm²

Area of face AEHD = Area of face BFGC = $(b \times h)$ cm²

Area of face ABFE = Area of face DHGC = $(l \times h)$ cm²

Total surface area (TSA) of cuboid = Sum of the areas of all its six faces

$= 2(l \times b) + 2(b \times h) + 2(l \times h)$

TSA (cuboid)= $2(lb + bh + lh)$

Lateral Surface Area of a Cuboid

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.

The lateral surface area of the cuboid

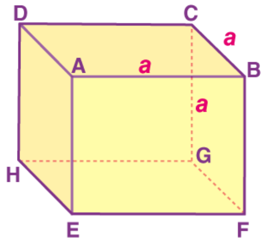
= Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC

$= 2(b \times h) + 2(l \times h)$

LSA (cuboid) = $2h(l + b)$

Cube

A cuboid whose length, breadth and height are all equal, is called a cube. It is a three-dimensional shape bounded by six equal squares. It has 12 edges and 8 vertices.



Total Surface Area of a cube

For cube, length = breadth = height

Suppose the length of an edge = a

Total surface area (TSA) of the cube = $2(a \times a + a \times a + a \times a)$

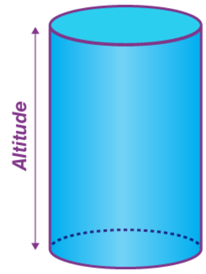
TSA (cube) = $2 \times (3a^2) = 6a^2$

Lateral Surface area of a cube

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.

Lateral surface area of cube = $2(a \times a + a \times a) = 4a^2$

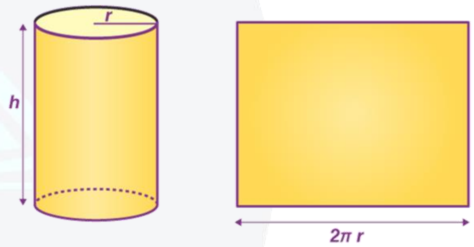
Right Circular Cylinder



A right circular cylinder is a closed solid that has two parallel circular bases connected by a curved surface in which the two bases are exactly over each other and the axis is at right angles to the base.

Curved Surface area of a right circular cylinder

Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter ($d = 2r$) of the circular base will be transformed into a rectangle of length $2\pi r$ and height h units. Thus,



Curved surface area (CSA) of a cylinder of base radius r and height $h = 2\pi \times r \times h$

Total surface area of a right circular cylinder

Total surface area (TSA) of a cylinder of base radius r and height $h = 2\pi \times r \times h$ + area of two circular bases

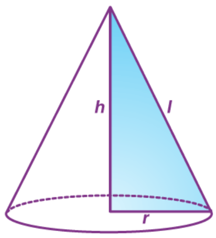
$\Rightarrow$ TSA = $2\pi \times r \times h + 2 \times \pi r^2$

$\Rightarrow$ TSA = $2\pi r (h + r)$

Right Circular Cone

A right circular cone is a circular cone whose axis is perpendicular to its base.

Relation between slant height and height of a right circular cone



The relationship between slant height(l) and height(h) of a right circular cone is:

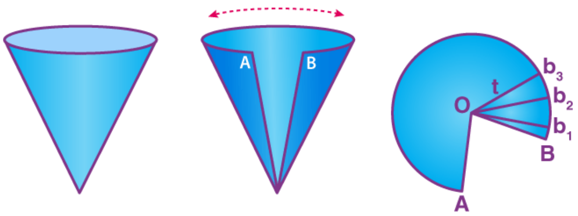
$l^2 = h^2 + r^2$ (Using Pythagoras Theorem)

Where r is the radius of the base of the cone.

Curved Surface Area of a Right Circular Cone

Consider a right circular cone with slant length l and radius r.

If a perpendicular cut is made from a point on the circumference of the base to the vertex and the cone is opened up, a sector of a circle with radius l is produced as shown in the figure below:



Label A and B and corresponding $b_1, b_2 \ldots b_n$ at equal intervals, with O as the common vertex. The Curved surface area (CSA) of the cone will be the sum of areas of the small triangles: $1/2 \times (b_1 + b_2 \ldots\ldots b_n) \times l$

$(b_1 + b_2 \ldots b_n)$ is also equal to the circumference of base = $2\pi r$

CSA of right circular cone = $(1/2) \times (2\pi r) \times l = \pi rl$ (On substituting the values)

Total Surface Area of a Right Circular Cone

Total surface area (TSA) = Curved surface area (CSA) + area of base = $\pi rl + \pi r^2 = \pi r(l + r)$

Sphere

A sphere is a closed three-dimensional solid figure, where all the points on the surface of the sphere are equidistant from the common fixed point called “centre”. The equidistant is called the “radius”.

Surface area of a Sphere

The surface area of a sphere of radius r = 4 times the area of a circle of radius r = $4 \times (\pi r^2)$

For a sphere Curved surface area (CSA) = Total Surface area (TSA) = $4\pi r^2$

Surface Area Formulas

Shapes	Surface Areas
Cuboid	2(lb + bh + hl)
Cube	$6a^2$
Right Circular Cylinder	$2\pi r(r + h)$
Right Circular Cone	$\pi r(l + r)$, $(l^2 = h^2 + r^2)$
Sphere	$4\pi r^2$

Volume of a Cuboid

The volume of a cuboid is the product of its dimensions.

Volume of a cuboid = length × breadth × height= lbh

Where l is the length of the cuboid, b is the breadth, and h is the height of the cuboid.

Volume of a Cube

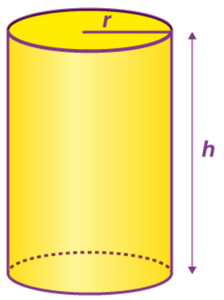
The volume of a cube = base area × height.

Since all dimensions are identical, the volume of the cube = a^3

Where a is the length of the edge of the cube.

Volume of a Right Circular Cylinder

The volume of a right circular cylinder is equal to base area × its height.



The volume of cylinder =πr2h

Where r is the radius of the base of the cylinder and h is the height of the cylinder.

Volume of a Right Circular Cone

The volume of a Right circular cone is 1/3 times the volume of a cylinder with the same radius and height. In other words, three cones make one cylinder of the same height and base.

The volume of right circular cone = $(1/3)\ \pi r^2 h$

Where r is the radius of the base of the cone and h is the height of the cone.

Volume of a Sphere

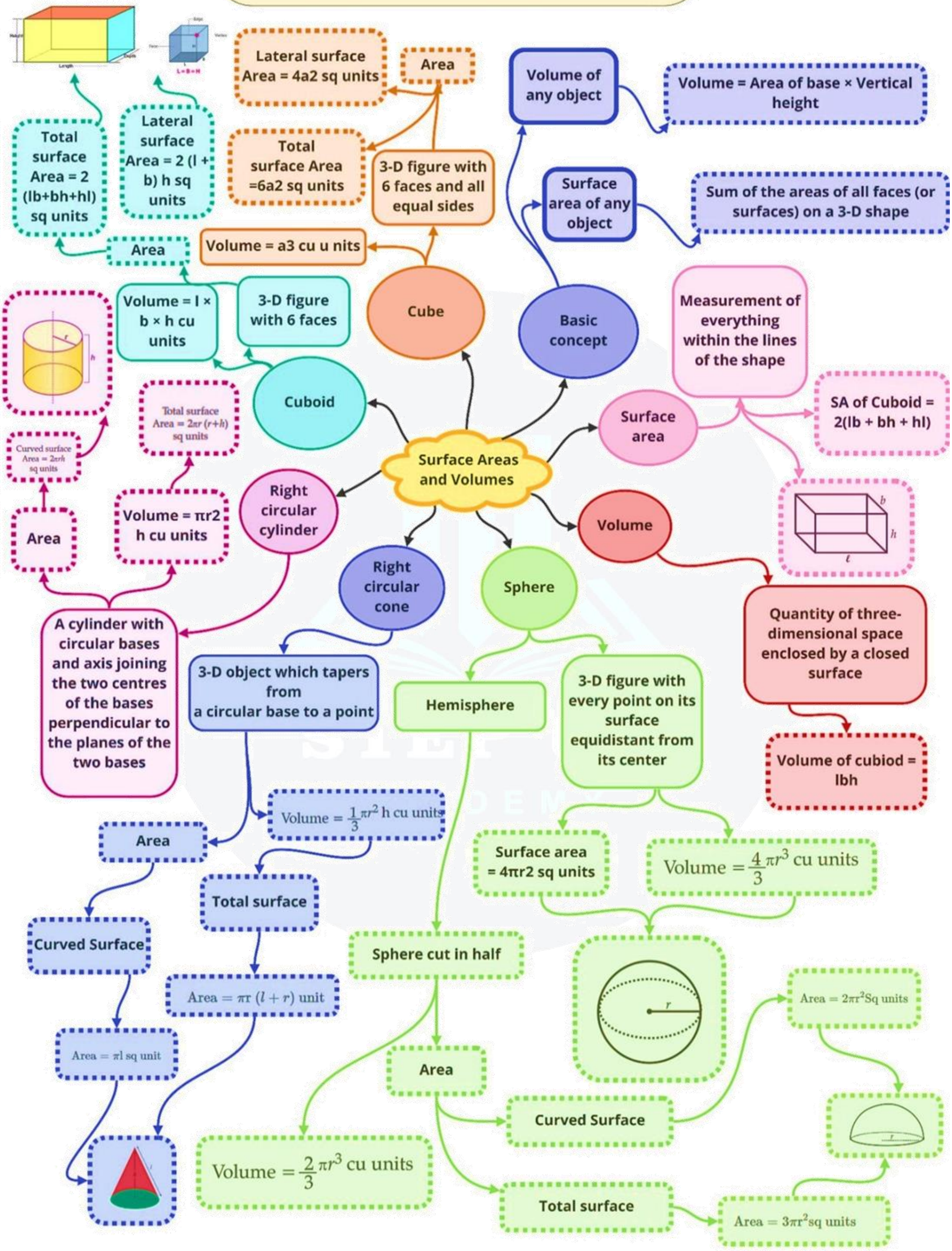
The volume of a sphere of radius r = $(4/3)\pi r^3$

Shapes	Volumes
Cuboid	length × breadth × height
Cube	a^3
Right Circular Cylinder	$\pi r^2 h$
Right Circular Cone	$\frac{1}{3} \pi r^2 h$
Sphere	$4/3\ \pi r^3$

Volume and Capacity

The volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. The unit of measurement of either volume or capacity is a cubic unit.

Class : 9th mathematics
Chapter- 13: Surface Areas and Volumes



Important Questions

Multiple Choice Questions

1. If the perimeter of one of the faces of a cube is 40cm, then its volume is:
   (a) $6000cm^3$
   (b) $1600cm^3$
   (c) $1000cm^3$
   (d) $600cm^3$
2. A cuboid having surface areas of 3 adjacent faces as a, b and c has the volume:
   (a) $3\sqrt{abc}$
   (b) $\sqrt{abc}$
   (c) ABC
   (d) $(ABC)^2$
3. The radius of a cylinder is doubled, and the height remains the same. The ratio between the volumes of the new cylinder and the original cylinder is
   (a) 1 : 2
   (b) 3 : 1
   (c) 4 : 1
   (d) 1 : 8
4. Length of diagonals of a cube of side a cm is
   (a) $\sqrt{2}a$ cm
   (b) $\sqrt{3}a$ cm
   (c) $\sqrt{3a}$ cm
   (d) 1 cm
5. Volume of spherical shell is
   (a) $2/3\ \pi r^3$
   (b) $3/4\ \pi r^3$
   (c) $4/3\ \pi(R^3 - r^3)$
   (d) None of these
6. Volume of hollow cylinder
   (a) $\pi(R^2 - r^2)h$
   (b) πR^2h
   (c) πr^2h
   (d) $\pi r^2(h_1 - h_1)$
7. The radius of a sphere is 2r, then its volume will be
   (a) $4/3\ \pi r^3$
   (b) $4\ \pi r^3$
   (c) $8/3\ \pi r^3$
   (d) $32/3\ \pi r^3$
8. In a cylinder, radius is doubled, and height is halved, curved surface area will be
   (a) Halved
   (b) Doubled
   (c) Same
   (d) Four time
9. The total surface area of a cone whose radius is r^2 and slant height 2l is
   (a) $2\pi r(l + r)$
   (b) $\pi r\left(l+\frac{r}{4}\right)$
   (c) $\pi r(l + r)$
   (d) $2\pi rl$
10. The radius of a hemispherical balloon increases from 6cm to 12cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is
   (a) 1 : 4
   (b) 1 : 3
   (c) 2 : 3
   (d) 2 : 1

Very Short Questions:

1. How much ice-cream can be put into a cone with base radius 3.5cm and height 12cm?
2. Calculate the edge of the cube if its volume is $1331cm^3$.
3. The curved surface area of a cone is 12320 sq. cm, if the radius of its base is 56cm, find its height.
4. Two cubes of edge 6cm are joined to form a cuboid. Find the total surface area of the cuboid.
5. A metallic sphere is of radius 4.9cm. If the density of the metal is 7.8 g/cm^2, find the mass of the sphere ($\pi = 22/7$)
6. The volume of a solid hemisphere is 1152 π cm^3. Find its curved surface area.
7. Find the diameter of a cylinder whose height is 5cm and numerical value of volume is equal to numerical value of curved surface area.
8. In a cylinder, if radius is halved and height is doubled, then find the volume with respect to original volume.

Short Questions:

1. A spherical ball is divided into two equal halves. If the curved surface area of each half is 56.57cm, find the volume of the spherical ball. [use π=3.14]

2. Find the capacity in liters of a conical vessel having height 8 cm and slant height 10cm.
3. Calculate the surface area of a hemispherical dome of a temple with radius 14m to be whitewashed from outside
4. A rectangular piece of paper is 22cm long and 10cm wide. A cylinder is formed by rolling the paper along its length. Find the volume of the cylinder.
5. A heap of wheat is in the form of a cone whose diameter is 10.5m and height is 3m. Find it volume. If $1m^3$ wheat cost is ₹ 10, then find total cost.
6. A cylindrical vessel can hold 154 g of water. If the radius of its base is 3.5cm, and $1cm^3$ of water weighs 1 g, find the depth of water.

Long Questions:

1. It costs ₹ 3300 to paint the inner curved surface of a 10m deep well. If the rate cost of painting is of ₹ 30 per m^2, find:

   (a) inner curved surface area

   (b) diameter of the well

   (c) capacity of the well.
2. Using clay, Anant made a right circular cone of height 48cm and base radius 12cm. Varsha reshapes it in the form of a sphere. Find the radius and curved surface area of the sphere so formed.
3. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of ₹ 498.96. If the rate of whitewashing is ₹ 4 per square metre, find the:

   (i) Inside surface area of the dome

   (ii) Volume of the air inside the dome.
4. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 5cm. Find the volume of the solid so obtained. If it is now revolved about the side 12cm, then what would be the ratio of the volumes of the two solids obtained in two cases?
5. A right triangle of hypotenuse 13cm and one of its sides 12cm is made to revolve taking side 12cm as its axis. Find the volume and curved surface area of the solid so formed.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

   c) Assertion is correct statement but reason is wrong statement.

   d) Assertion is wrong statement but reason is correct statement.

   Assertion: If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

   Reason: Curved surface area is increased when diameter decreases.
2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

   c) Assertion is correct statement but reason is wrong statement.

   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The external dimensions of a wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of the wood is 15 mm, then the internal volume is $765 cm^3$.

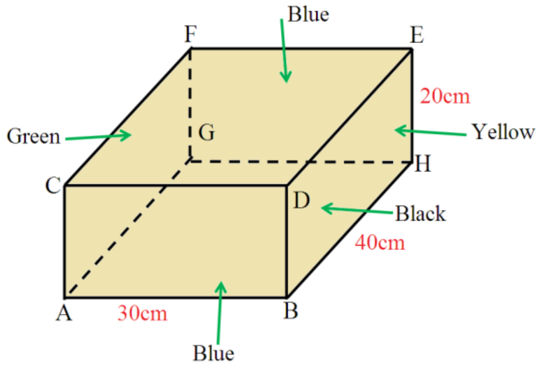
   Reason: If external dimensions of a rectangular box be l, b and h and the thickness of its sides be x, then its internal volume is $(l - 2x)(b - 2x)(h - 2x)$.

Case Study Questions:

1. Read the Source/ Text given below and answer these questions:

   Veena planned to make a jewellery box to gift her friend Reeta on her marriage. She made the jewellery box of wood in the shape of a cuboid. The jewellery box has the dimensions as shown in the figure below. The rate of painting the exterior of the box is Rs. 2 per cm2. After making the box she took help from his friends to decorate the box.

The blue colour was painted by Deepak, Black by Suresh, green by Harsh and the yellow was painted by Naresh.



i. What is the volume of the box?
   a. 24000cm^3
   b. 1200cm^3
   c. 800cm^3
   d. 600cm^3

ii. How much area did Suresh paint?
   a. 24000cm^2
   b. 1200cm^2
   c. 800cm^2
   d. 600cm^2

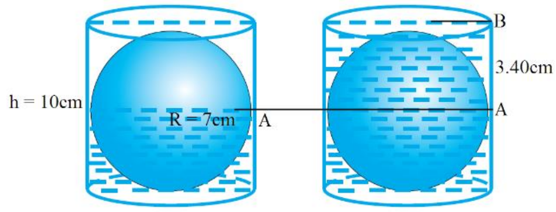
iii. How much area did Deepak paint?
   a. 24000cm^2
   b. 600cm^2
   c. 800cm^2
   d. 1200cm^2

iv. What amount did Harsh charge?
   a. Rs. 800
   b. Rs. 1200
   c. Rs. 1600
   d. Rs. 2000

v. What amount did Veena pay for painting:
   a. Rs. 2600
   b. Rs. 5200
   c. Rs. 5000
   d. Rs. 6000

2. Read the passage given below and answer these questions:

Dev was doing an experiment to find the radius r of a sphere. For this he took a cylindrical container with radius R = 7cm and height 10cm. He filled the container almost half by water as shown in the left figure. Now he dropped the yellow sphere in the container. Now he observed as shown in the right figure the water level in the container raised from A to B equal to 3.40cm.



i. What is the approximate radius of the sphere?
   a. 7cm
   b. 5cm
   c. 4cm
   d. 3cm

ii. What is the volume of the cylinder?
   a. 700cm^3
   b. 500cm^3
   c. 1540cm^3
   d. 2000cm^3

iii. What is the volume of the sphere?
   a. 700cm^3
   b. 600cm^3
   c. 500cm^3
   d. 523.8cm^3

iv. How many litres water can be filled in the full container? (Take 1 litre = 1000cm^3):
   a. 1.50
   b. 1.44
   c. 1.54
   d. 2

v. What is the surface area of the sphere?
   a. 314.3m^2
   b. 300m^2
   c. 400m^2
   d. 350m^2

Answer Key

Multiple Choice Questions

1. (c) $1000 cm^3$
2. (b) $\sqrt{abc}$
3. (c) 4 : 1
4. (b) $\sqrt{3}a$ cm
5. (c) $4/3\ \pi(R^3 - r^3)$
6. (a) $\pi(R^2 - r^2)h$
7. (d) $32/3\ \pi r^3$
8. (c) Same
9. (b) $\pi r\left(l+\frac{r}{4}\right)$
10. (a) 1 : 4

Very Short Answer:

1. Here, radius (r) = 3.5cm and height (h) = 12cm

   ∴ Amount of ice-cream = $\frac{1}{3}\pi r^2$

   $= \frac{1}{3}\times\frac{22}{7} \times 3.5 \times 3.5 \times 12$

   $= 154\ cm^3$

2. Volume of cube = $1331\ cm^3$

   $(Side)^3 = 1331$

   Side = $(11 \times 11 \times 11)\ \frac{1}{3} = 11cm$

3. Here, radius of base of a cone (r) = 56 cm

   And, curved surface area = $12320\ cm^2$

   $\pi rl = 12320$

   $l = \frac{12320}{\pi r}$

   $= \frac{12320\times7}{22\times56} = 70$ cm

   Again, we have

   $r^2 + h^2 = l^2$

   $h^2 = l^2 - r^2 = 70^2 - 56^2$

   $= 4900 - 3136 = 1764$

   $h = \sqrt{1764} = 42cm$

   Hence, the height of the cone is 42 cm.

4. When two cubes are joined end to end, then

   Length of the cuboid = 6 + 6 = 12 cm

   Breadth of the cuboid = 6 cm

   Height of the cuboid = 6 cm

   Total surface area of the cuboid = 2 (lb + bh + hl)

   $= 2(12 \times 6 + 6 \times 6 + 6 \times 12)$

   $= 2(72 + 36 + 72) = 2(180)$

   $= 360\ cm^2$

5. Here, radius of metallic sphere (r) = 4.9cm

   ∴ Volume of metallic sphere = $\frac{4}{3}\pi r^3$

   $= \frac{4}{3}\times\frac{22}{7}\times(4.9)^3$

   $= \frac{4}{3}\times\frac{22}{7}\times4.9\times4.9\times4.9$

   $= 493\ cm^3$

   Density of metal used = $7.8\ g/cm^3$

   Mass of the sphere = Volume × Density

   = 493 × 7.8 = 3845.4 g

6. Here, volume of hemisphere = $1152\ \pi\ cm^3$

   $\therefore \frac{2}{3}\pi r^3 = 1152$

   $\Rightarrow r^3 = (12)^3\ \pi$

   $\Rightarrow r\ \frac{1152\times3}{3} = 1728$

   $\Rightarrow r^3 = (12)^3$

   Now, curved surface area = $2\pi r^2$

   $= 2 \times \pi \times (12)^2 = 288\pi\ cm^2$

7. Here, height of cylinder (h) = 5cm

   According to the statement of the question, we have

   $\pi r^2h = 2\pi rh$

   r = 2cm

   Thus, diameter of the base of the cylinder is 2 × 2 i.e., 4cm.

8. Here, $r = \frac{\pi}{2}\cdot\ h = 2h$

   ∴ Volume of cylinder = $\pi\left(\frac{r}{2}\right)^2 2h = \frac{1}{2}\pi r^2h$

   Original volume of cylinder = πr^2h

   Volume w.r.t. original volume of cylinder

   $= \frac{\frac{1}{2}\pi r^2h}{\pi r^2h} = \frac{1}{2}$

Short Answer:

1. Since curved surface of half of the spherical ball = 56.57 cm^2

$2\pi r^2 = 56.57$

$\Rightarrow r^2 = \frac{56.57}{2\times3.14} = 9$

$\Rightarrow r = 3$ cm

Now, volume of spherical ball = $\frac{4}{3}\pi r^3$

$\therefore \frac{4}{3} \times 3.14 \times 3 \times 3 \times 3$

$= 113.04$ cm^3

2. Height of conical vessel (h) = 8cm

Slant height of conical vessel (l) = 10cm

$\therefore r^2 + h^2 = l^2$

$\Rightarrow r^2 + 8^2 = 10^2$

$\Rightarrow r^2 = 100 - 64 = 36$

$\Rightarrow r = 6$cm

Now, volume of conical vessel = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7} \times 6 \times 8 = 301.71cm^3 = 0.30171$ litre

3. Here, radius of hemispherical dome (r) = 14m

Surface area of dome = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 14 = 1232$m2

Hence, total surface area to be whitewashed from outside is 1232m2.

4. Since rectangular piece of paper is rolled along its length.

$\therefore 2\pi r = 22$

$r = \frac{22\times7}{2\times22} = 3.5$ cm

Height of cylinder (h) = 10cm

$\therefore$ Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 10 = 385$ cm^3

5. Diameter of cone = 10.5 m

Radius of cone (r) = 5.25 m

Height of cone (h) = 3m

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7} \times 5.25 \times 5.25 \times 3$

$= 86.625$ m^3

Cost of 1m^3 of wheat = ₹ 10

Cost of 86.625 m^3 of wheat = ₹ 10 × 86.625

= ₹ 866.25

6. Since 1 cm^3 of water weighs 1 g.

$\therefore$ Volume of cylindrical vessel = 154 cm^3

$\pi r^2 h = 154$

$\frac{22}{7} \times 3.5 \times 3.5 \times h = 154$

$h = \frac{154\times7}{22\times3.5\times3.5}$

h = 4 cm

Hence, the depth of water is 4cm.

Long Answer:

1. Depth of well (h) = 10m

Cost of painting inner curved surface is ₹ 30 per m^2 and total cost is ₹ 3300

$\therefore$ Curved surface area = $\frac{3300}{30} = 110$ m^2

$2\pi rh = 110$

$r = \frac{110}{2\pi h}$

$= \frac{110\times7}{2\times22\times10}$

= 1.75 m = 175 cm

Now, volume of the well = $\pi r^2 h$

$= \frac{22}{7} \times 1.75 \times 1.75 \times 10 = 96.25$ m^3

Hence, inner curved surface area is 110m^2, diameter of the well is 2×1.75 i.e., 3.5m and capacity of the well is 96.25 m^3.

2. Height of cone (h) = 48 cm

Radius of the base of cone = 12 cm

Let R be the radius of sphere so formed

$\therefore$ Volume of sphere = Volume of cone

$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$

$4R^3 = 12 \times 12 \times 48$

$R^3 = 12 \times 12 \times 12$

R = 12 cm

Now, curved surface area of sphere = 4πR2

$= 4 \times \frac{22}{7} \times 12 \times 12$

= 1810.29 cm

3. Here, dome of building is a hemisphere.

Total cost of whitewashing inside the dome

= ₹ 498.96

Rate of whitewashing = ₹ 4 per m^2

∴ Inside surface area of the dome = $\frac{498.96}{4}$

= 124.74 m^2

∴ $2\pi r^2 = 124.74$

$2 \times \frac{22}{7} \times r^2 = 124.74$

$r^2 = \frac{124.74 \times 7}{2 \times 22}$

⇒ $r^2 = 19.845$

⇒ $r = 4.45$ cm

Volume of the air inside the dome = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 4.45 \times 4.45 \times 4.45$

= 184.63 cm^3

4. Here, right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 5cm.

∴ Radius of the base of cone = 12cm

Height of the cone = 5cm

∴ Volume of the cone = $\frac{1}{3}\pi(12)^2(5) = \frac{\pi}{3} \times 720 cm^3$

Again, right triangle ABC is now revolved about the side 12 cm.

∴ Radius of the base of cone = 5 cm

Height of the cone = 12 cm

∴ Volume of the cone = $\frac{1}{3}\pi(5)^2(12) = \frac{\pi}{3} \times 300 cm^3$

Now, the required ratio of their volumes

$= \frac{\pi}{3} \times 720 : \frac{\pi}{3} \times 300$

= 12 : 5

5. Here, hypotenuse and one side of a right triangle are 13cm and 12cm respectively

∴ Third side = $\sqrt{(13)^2 - (12)^2}$

$= \sqrt{169 - 144}$

$= \sqrt{25} = 5$ cm

Now, given triangle is revolved, taking 12cm as its axis

∴ Radius of the cone (r) = 5cm

Height of the cone (h) = 12cm

Slant height of the cone (l) = 13cm

∴ Curved surface area = $\pi rl = \pi(5)(13) = 65\pi cm^2$

Volume of the cone = $\frac{1}{2}\pi r^2 h = \frac{1}{2}\pi \times 5 \times 5 \times 12$

= 100π cm^3

Hence, the volume and curved surface area of the solid so formed are 100π cm^3 and 65π cm^2 respectively.

Assertion and Reason Answers-

1. (c) Assertion is correct statement but reason is wrong statement.

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Case Study Questions:

1.

(i)	(a)	24000cm^3
(ii)	(b)	1200cm^2
(iii)	(d)	1200cm^2
(iv)	(c)	Rs. 1600
(v)	(b)	Rs. 5200

2.

(i)	(b)	5cm
(ii)	(c)	1540cm^3
(iii)	(d)	523.8cm^3
(iv)	(c)	1.54
(v)	(a)	314.3m^2

Statistics 14

Statistics

1. Facts or figures collected with a definite purpose are called **data**.
2. **Statistics** deals with collection, presentation, analysis and interpretation of numerical data.
3. Arranging data in an order to study their salient features is called **presentation of data**.
4. Data arranged in ascending or descending order is called arrayed data or an **array**.
5. When an investigator with a definite plan or design in mind collects data first handedly, it is called primary data.
6. Data when collected by someone else, say an agency or an investigator, comes to you, is known as the secondary data.
7. **Range** of the data is the difference between the maximum and the minimum values of the observations.
8. The small groups obtained on dividing all the observations are called classes or **class intervals** and the size is called the **class size** or class width.

   Class size = Upper limit – Lower limit
9. **Class mark** of a class is the mid-value of the two limits of that class.
10. The number of times an observation occurs in the data is called the **frequency** of the observation.
11. A **frequency distribution** in which the upper limit of one class differs from the lower limit of the succeeding class is called an Inclusive or discontinuous frequency distribution.
12. A frequency distribution in which the upper limit of one class coincides with the lower limit of the succeeding class is called an exclusive or **continuous frequency distribution**.
13. In case of continuous frequency distribution, the upper limit of a class is not to be included in that class while in discontinuous both the limits are included.
14. The **cumulative frequency** of a class-interval is the sum of frequencies of that class and the classes which precede (come before) it.
15. A data can be represented graphically through:

    (i) Bar graph (ii) Histogram (iii) Frequency polygon.
16. A **bar graph** is a diagram showing a system of connections or interrelations between two or more things by using bars.
17. In a bar graph, rectangular bars of uniform width are drawn with equal spacing between them on one axis, usually the x-axis. The value of the variable is shown on the other axis that is the y-axis.
18. A **histogram** is a graphical representation of a frequency distribution in the form of rectangles with class intervals as bases and heights proportional to the corresponding frequencies such that there is no gap between any two successive rectangles.
19. If classes are not of equal width, then the height of the rectangle is calculated by the ratio of the frequency of that class, to the width of that class
20. **Frequency polygons** are a graphical device for understanding the shapes of distributions.

21. If both a histogram and a frequency polygon are to be drawn on the same graph, then we should first draw the histogram and then join the mid-points of the tops of the adjacent rectangles in the histogram with line-segments to get the frequency polygon.
22. A measure of central tendency tries to estimate the central value which represents the entire data.
23. The three **measures of central tendency** for ungrouped data are mean, mode and median.
24. The **mean** value of a variable is defined as the sum of all the values of the variable divided by the number of values.
25. If $x_1, x_2, x_3, \ldots\ldots, x_n$ are n values of a variable X, then the arithmetic mean of these values is given by:

$$\text{Mean } (\bar{x}) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

If a variate X takes values $x_1, x_2, x_3, \ldots, x_n$ with corresponding frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively, then arithmetic mean of these values is given by

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

26. **Median** is the value of middle most observation(s).
27. The median is calculated only after arranging the data in ascending order or descending order.

$$\begin{cases} \text{If n is odd, then median } = \left(\frac{n+1}{2}\right)^{th} \text{ observation} \\ \text{If n is even, then median} = \left\{ \dfrac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2} \right\} \end{cases}$$

28. **Mode** of a statistical data is the value of that variate which has the maximum frequency.
29. The variate corresponding to the highest frequency is to be taken as the mode and not the frequency.
30. The disadvantage of arithmetic mean is that it is affected by extreme values.
31. The disadvantage of mode is that it is not uniquely defined in many cases.

Introduction to Statistics

A study dealing with the collection, presentation and interpretation and analysis of data is called as statistics.

Data

- Facts /figures numerical or otherwise collected for a definite purpose is called as data.
- data collected first-hand data:- Primary
- Secondary data: Data collected from a source that already had data stored

Frequency

The number of times a particular instance occurs is called frequency in statistics.

Ungrouped data

Ungrouped data is data in its original or raw form. The observations are not classified in groups.

Grouped data

In grouped data, observations are organized in groups.

Class Interval

- The size of the class into which a particular data is divided.
- E.g divisions on a histogram or bar graph.
- Class width = upper class limit – lower class limit

Regular and Irregular class interval

Regular class interval: When the class intervals are equal or of the same sizes.

E.g 0-10, 10-20, 20-30….. 90-100

Irregular class interval: When the class intervals are of varying sizes.

E.g 0-35, 35-45, 45-55, 55- 80, 80-90, 90-95, 95-100

Frequency table

A frequency table or distribution shows the occurrence of a particular variable in a tabular form.

Sorting

- Raw data needs to be sorted in order to carry out operations.
- Sorting ⇒ ascending order or descending order

Ungrouped frequency table

When the frequency of each class interval is not arranged or organised in any manner.

Grouped frequency table

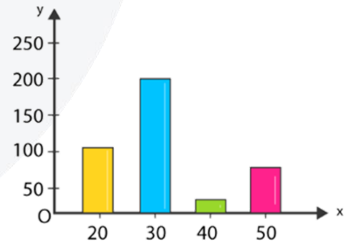
The frequencies of the corresponding class intervals are organised or arranged in a particular manner, either ascending or descending.

Graphical Representation of Data

Graphical representation of data using bars of equal width and equal spacing between them (on one axis). The height

Savings (in percentage)	Number of Employees (Frequency)
20	105
30	199
40	29
50	73
Total	400

The data can be represented as:



Variable being a number

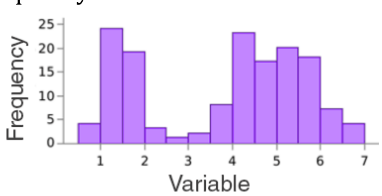
- A variable can be a number such as 'no. of students' or 'no. of months'.
- Can be represented by bar graphs or histograms depending on the type of data.

Discrete → bar graphs

Continuous → Histograms

Histograms

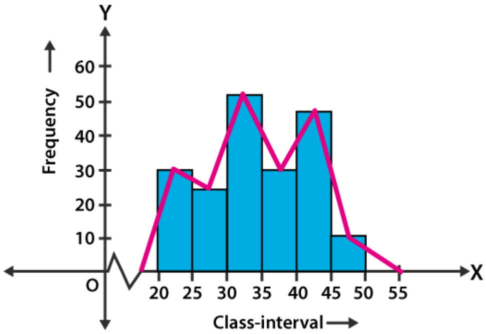
- Like bar graphs, but for continuous class intervals.
- Area of each rectangle is ∝ Frequency of a variable and the width is equal to the class interval.



Frequency polygon

If the midpoints of each rectangle in a histogram are joined by line segments, the figure formed will be a frequency polygon.

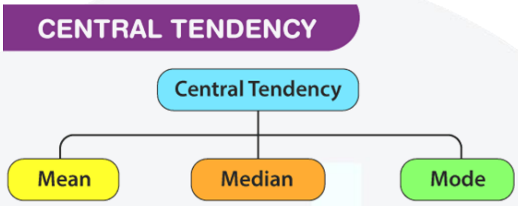
Can be drawn without histogram. Need midpoints of class intervals



Midpoint of class interval

The midpoint of the class interval is called a class mark

Class mark = (Upper limit + Lower limit)/2



Equality of areas

Addition of two class intervals with zero frequency preceding the lowest class and succeeding the highest-class intervals enables to equate the area of the frequency polygon to that of the histogram (Using congruent triangles.)

Measures of Central Tendency

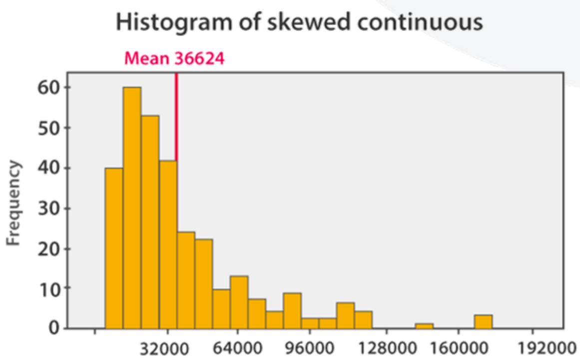
Average

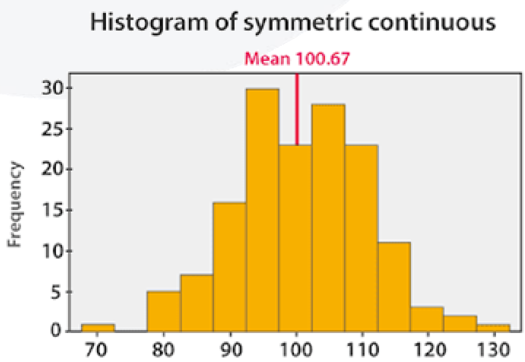
The average of a number of observations is the sum of the values of all the observations divided by the total number of observations.

Mean

Mean for ungrouped frequency distribution, $\bar{x} = \frac{\sum x_i f_i}{f_i}$

Where f_i is the frequency of i^{th} observation x_i





Mode

- The most frequently occurring observation is called the mode.
- The class interval with the highest frequency is the modal class.

Mode
5
5
5
4
4
3
2
2
1

Median

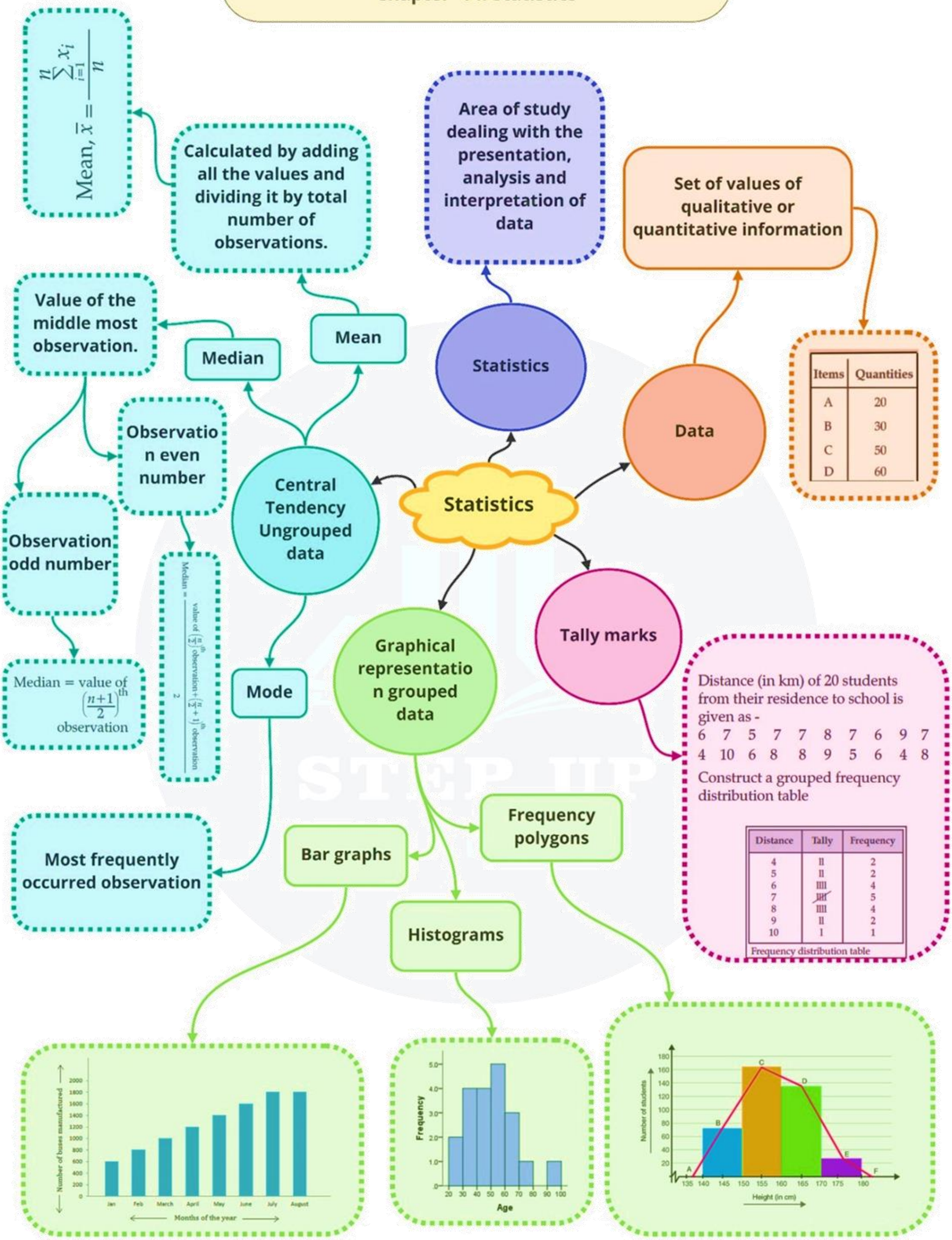
- Value of the middlemost observation.
- If n(number of observations) is odd, Median =[(n+1)/2]th observation.
- If n is even, the Median is the mean or average of (n/2)th and [(n+1)/2]th observation.

Median odd
23
21
18
16
15
13
12
10
9
7
6
5
2

Median even
40
38
35
33
32
30
29
27
26
24
23
22
19
17

28

Class : 9th mathematics
Chapter- 14: Statistics



Important Questions

Multiple Choice Questions

1. The class mark of the class 90-130 is:
   (a) 90
   (b) 105
   (c) 115
   (d) 110
2. The range of the data:
   25, 81, 20, 22, 16, 6, 17,15,12, 30, 32, 10, 91, 8, 11, 20 is
   (a) 10
   (b) 75
   (c) 85
   (d) 26
3. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The upper limit of the class is:
   (a) 6
   (b) 7
   (c) 10
   (d) 13
4. The width of each of five continuous classes in a frequency distribution is 5 and the lower class-limit of the lowest class is 10. The lower class-limit of the highest class is:
   (a) 15
   (b) 30
   (c) 35
   (d) 40
5. Let m be the mid-point and 1 be the lower-class limit of a class in a continuous frequency distribution. The upper-class limit of the class is:
   (a) 2m + l
   (b) 2m – l
   (c) m – l
   (d) m – 2l
6. The class marks of a frequency distribution are given as follows:
   15, 20, 25, ...
   The class corresponding to the class mark 15 is:
   (a) 12.5 – 17.5
   (b) 17.5 – 22.5
   (c) 18.5 – 21.5
   (d) 19.5 – 20.5
7. In the class intervals 10-20, 20-30, the number 20 is included in:
   (a) 10-20
   (b) 20-30
   (c) Both the intervals
   (d) None of these intervals
8. A grouped frequency table with class intervals of equal sizes using 250-270 (270 not included in this interval) as one of the class interval is constructed for the following data:
   268, 220, 368, 258, 242, 310, 272, 342, 310, 290, 300, 320, 319, 304,402, 318, 406, 292, 354, 278, 210, 240, 330, 316, 406, 215, 258, 236.
   The frequency of the class 370-390 is:
   (a) 0
   (b) 1
   (c) 3
   (d) 5
9. A grouped frequency distribution table with classes of equal sizes using 63-72 (72 included) as one of the class is constructed for the following data:
   30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 96, 102, 110, 88, 74, 112, 14, 34, 44.
   The number of classes in the distribution will be:
   (a) 9
   (b) 10
   (c) 11
   (d) 12
10. To draw a histogram to represent the following frequency distribution:

Class interval	5-10	10-15	15-25	25-45	45-75
Frequency	6	12	10	8	15

   the adjusted frequency for the class 25-45 is:
   (a) 6
   (b) 5
   (c) 3
   (d) 2

Very Short Questions:

1. The points scored by a basketball team in a series of matches are follows:
   17, 7, 10, 25, 5, 10, 18, 10 and 24. Find the range.

2. The points scored by a basketball team in a series of matches are as follows:
   17, 2, 7, 27, 25, 5, 14, 18, 10. Find the median.
3. The scores of an English test (out of 100) of 20 students are given below:
   75, 69, 88, 55, 95, 88, 73, 64, 75, 98, 88, 95, 90, 95, 88, 44, 59, 67, 88, 99. Find the median and mode of the data.
4. Mean of 20 observations is 17. If in the observations, observation 40 is replaced by 12, find the new mean.
5. Mean of 36 observations is 12. One observation 47 was misread as 74. Find the correct mean.
6. The median of the data 26, 56, 32, 33, 60, 17, 34, 29, 45 is 33. If 26 is replaced by 62, then find the new median.
7. There are 50 numbers. Each number is subtracted from 53 and the mean of the numbers so obtained is found to be - 3.5. Find the mean of the given numbers
8. To draw a histogram to represent the following frequency distribution

Class interval	5-10	10-15	15-25	25-45	45-75
Frequency	6	12	10	8	15

Find the adjusted frequency for the class 25-45.

Short Questions:

1. For a particular year, following is the distribution of ages (in years) of primary school teachers in a district:

Age (in years)	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of Teachers	10	30	50	50	30	6	4

(i) Write the lower limit of first-class interval.

(ii) Determine the class limits of the fourth-class interval.

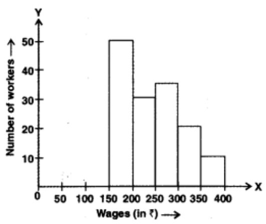
(iii) Find the class mark of the class 45 – 50.

(iv) Determine the class size.

2. Find the mean of the following distribution:

x	5	10	15	20	25
y	4	12	20	28	36

3. In figure, there is a histogram depicting daily wages of workers in a factory. Construct the frequency distribution table.



4. Ten observations 6, 14, 15, 17, x + 1, 2x – 13, 30, 32, 34, 43 are written in ascending order. The median of the data is 24. Find the value of x.
5. Draw a histogram for the given data:

Class Interval	Frequency
20 – 25	21
25 – 30	22
30 – 35	50
35 – 40	75
40 – 45	67
45 – 50	51
50 – 55	18

6. Given are the scores (out of 25) of 9 students in a Monday test:
   14, 25, 17, 22, 20, 19, 10, 8 and 23
   Find the mean score and median score of the data.

Long Questions:

1. Find the mean salary of 60 workers of a factory from the following table:

Salary (in ₹)	Number of Workers
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1
Total	60

2. In a school marks obtained by 80 students are given in the table. Draw a histogram. Also, make frequency polygon.

Marks obtained (Mid Value)	Number of students
305	12
315	18
325	28
335	15
345	5
355	2

3. The following two tables gives the distribution of students of two sections according to the marks obtained by them:

Section-A		Section-B	
Marks	Frequency	Marks	Frequency
0-10	3	0-10	5

10-20	9	10-20	19
20-30	17	20-30	15
30-40	12	30-40	10
40-50	9	40-50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

4. The mean weight of 60 students of a class is 52.75 kg. If mean weight of 25 students of this class is 51 kg, find the mean weight of remaining 35 students of the class.

5. Find the missing frequencies in the following frequency distribution. If it is known that the mean of the distribution is 50.16 and the total number of items is 125.

x	10	30	50	70	90
f	17	f_1	32	f_2	19

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The range of the first 6 multiples of 6 is 9.

   Reason: Range = Maximum value – Minimum value

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
   c) Assertion is correct statement but reason is wrong statement.
   d) Assertion is wrong statement but reason is correct statement.

   Assertion: The median of the following observation 0, 1, 2, 3, x, x + 2, 8, 9, 11, 12 arranged in ascending order is 63, then the value of x is 62.

   Reason: Median of n even observations is

$$\frac{\left(\frac{n}{2}\right)^{th}\text{term}+\left(\frac{n}{2}+1\right)^{th}\text{term}}{2}.$$

Answer Key

Multiple Choice Questions

1. (d) 110
2. (c) 85
3. (d) 13
4. (b) 30
5. (b) 2m – l
6. (a) 12.5 – 17.5
7. (b) 20-30
8. (a) 0
9. (b) 10
10. (d) 2

Very Short Answer:

1. Here, maximum points = 25 and

   minimum points = 5

   Range = Maximum value - Minimum value

   = 25 – 5 = 20

2. Here, points scored in ascending order are 2, 5, 7, 10, 14, 17, 18, 25, 27, we have n = 9 terms

   $\therefore$ Median = $\left(\frac{n+1}{2}\right)^{th}$ term = $\left(\frac{9+1}{2}\right)^{th}$ term

   = 5^{th} term = 14

3. The scores of an English test (out of 100) in ascending order are

44, 55, 59, 64, 67, 69, 73, 75, 75, 88, 88, 88, 88, 88, 90, 95, 95, 95, 98, 99

Here, n = 20

∴ Median = Mean of $\left(\frac{n}{2}\right)^{th}$ term & $\left(\frac{n}{2}+1\right)^{th}$ term

= Mean of 10^{th} and 11^{th} term Median

= Mean of 88 and 88 = 88

Mode = 88 [∵ 88 occurred max. no. of times i.e., 5 times]

4. Since mean of 20 observations is 17

∴ Sum of the 20 observations = 17 × 20 = 340

New sum of 20 observations = 340 - 40 + 12 = 312

New mean = $\frac{312}{20}$ = 15.6

5. Mean of 36 observations = 12

Total of 36 observations = 36 × 12 = 432

Correct sum of 36 observations = 432 – 74 + 47 = 405

Correct mean of 36 observations = $\frac{405}{36}$ = 11.25

6. Here, the given data in ascending order is 17, 29, 32, 33, 34, 45, 56, 60, 62

Now, median is $\left(\frac{9+1}{2}\right)^{th}$ term i.e., 5^{th} term.

Hence, new median is 34.

7. There are 50 numbers. Each number is subtracted from 53 and the mean of the numbers so obtained is found to be – 3.5. Find the mean of the given numbers.

Solution:

Let x be the mean of 50 numbers.

i. Sum of 50 numbers = 50x

Since each number is subtracted from 53.

According to question, we have

$$\frac{53\times50-50x}{50} = -3.5$$

⇒ 2650 – 50x = – 175 ⇒ 50x = 2825

$$\Rightarrow x = \frac{2825}{50} = 56.5$$

8. Adjusted frequency of a class

$$= \frac{\text{Minimum class size of frequency distribution} \times \text{Frequency of given class}}{\text{Class size of given class}}$$

∴ Adjusted frequency for the class

$$25\text{–}45 = \frac{5\times8}{20} = 2$$

Short Answer:

1. (i) First class interval is 15 – 20 and its lower limit is 15.

(ii) Fourth class interval is 30 – 35

Lower limit is 30 and upper limit is 35.

(iii) Class mark of the class 45 – 50 = (45 + 502)/2 = 95/2 = 47.5

(iv) Class size = Upper limit of each class interval – Lower limit of each class interval

∴ Here, class size = 20 – 15 = 5

2.

x	f	fx
5	4	20
10	12	120
15	20	300
20	28	560
25	36	900
Total	Σf = 100	Σfx = 1900

Now, mean $(\bar{x}) = \frac{\sum fx}{\sum f} = \frac{1900}{100} = 19$

3.

Class interval	**Frequency**
150-200	50
200-250	30
250-300	35
300-350	20
350-400	10
Total	145

4. Here, the arranged data is 6, 14, 15, 17, x + 1, 2x – 13, 30, 32, 34, 43

Total number of observations = 10

Here, 10 is an even number, therefore, median will be the mean of $\left(\frac{10}{2}\right)^{th}$ and $\left(\frac{10}{2}+1\right)^{th}$ observation.

$$\therefore \text{Median} = \frac{5^{th}\text{ observation} + 6^{th}\text{ observation}}{2}$$

$$= \frac{x+1+2x-13}{72} = \frac{3x-12}{2}$$

But median of data is 24 (given)

$$\Rightarrow \frac{3x-122}{2} = 24$$

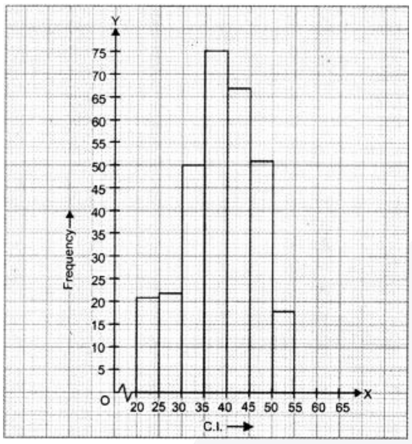
⇒ 3x – 12 = 48

⇒ 3x = 60

⇒ x = 20

∴ The value of x = 20

5. Let us represent class-intervals along x-axis and corresponding frequencies along y-axis on a suitable scale, the required histogram is as under:



6. Ascending order of scores is:

8, 10, 14, 17, 19, 20, 22, 23, 25

Now, mean score =

$$\frac{8+10+14+17+19+20+22+23+25}{9} = \frac{158}{9}$$

= 17.5 marks

Median = $\left(\frac{n}{2}+1\right)^{th}$ observation because n is odd

= $\left(\frac{9}{2}+1\right)^{th}$ observation = 5^{th} observation

= 19 marks

Long Answer:

1.

Salary (in ₹) (x_i)	Number of workers (f_i)	$f_i x_i$
3000	16	48000
4000	12	48000
5000	10	50000
6000	8	48000
7000	6	42000
8000	4	32000
9000	3	27000
10000	1	10000
Total	$\Sigma f_i = N = 60$	$\Sigma f_i x_i = 305000$

∴ Mean $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

⇒ $\bar{x} = \frac{305000}{60}$ ⇒ $\bar{x} = 5083.33$

Hence, mean salary of 60 workers is ₹ 5083.33

2. ∴ Lower limit of first-class interval is

305 – 102 = 300

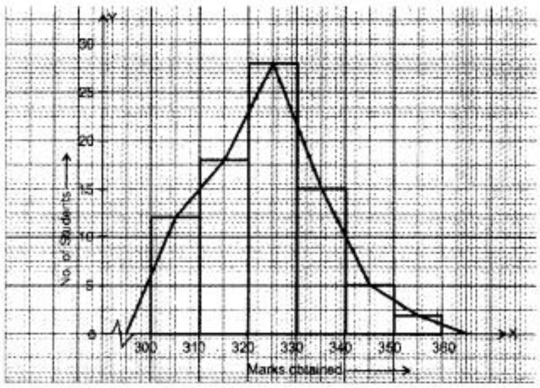
Upper limit of first-class interval is

305 + 102 = 310

Thus, first class interval is 300 – 310

Marks obtained	Number of students
300-310	12
310-320	18
320-330	28
330-340	15
340-350	5
350-360	2

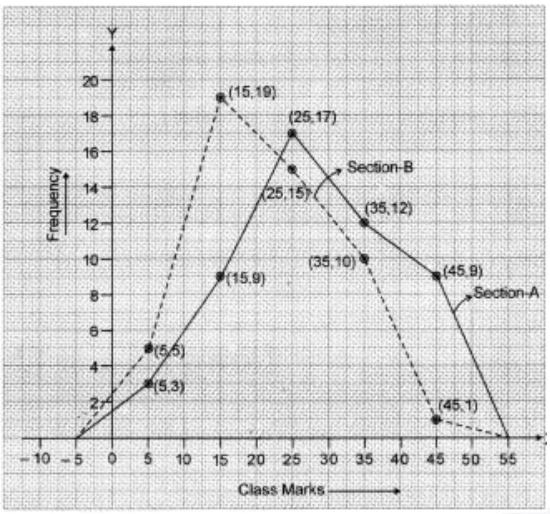
Required histogram and frequency polygon is given on the graph paper



3. The class marks are as under:

Marks	Class Marks	Section-A Frequency	Section-B Frequency
0-10	5	3	5
10-20	15	9	19
20-30	25	17	15
30-40	35	12	10
40-50	45	9	1

Let us take class marks on X-axis and frequencies on Y-axis. To plot frequency polygon of Section-A, we plot the points (5, 3), (15, 9), (25,17), (35,12), (45, 9) and join these points by (15,19). line segments. To plot frequency polygon of Section-B, we plot the points (5, 5), (15, 19), (25, 15), (35, 10), (45, 1) on the same scale and join these points by dotted line segments.



From the above two polygons, clearly the performance of Section A is better.

4. Total weight of 60 students = 60 × 52.75kg = 3165kg

Total weight of 25 students = 25 × 51kg = 1275kg

∴ Total weight of 35 students = (3165 – 1275) kg = 1890kg

∴ Mean weight of 35 students = 189035 = 54kg

5. Since total number of items = 125

∴ $17 + f_1 + 32 + f_1 + 19 = 125$

$f_1 + f_2 = 125 - 17 - 32 - 19$

$f_1 + f_2 = 57$ .....(i)

Now, mean of data = 50.16

We know that

$$\frac{\sum f_i x_i}{\sum f_i} = 50.16$$

$$\Rightarrow \frac{10\times17+30\times f_1+50\times32+70\times f_2+90\times19}{125}$$

= 50.16

$\Rightarrow 170 + 30f_1 + 1600 + 70f_2 + 1710 = 125\times50.16$

$\Rightarrow 3480 + 30f_1 + 70f_2 = 6270$

$\Rightarrow 30f_1 + 70f_2 = 6270 - 3480$

$\Rightarrow 30f_1 + 70f_2 = 2790$

$\Rightarrow 3f_1 + 7f_2 = 279$ ... (ii)

Multiplying (i) by 3, we have

$3f_1 + 3f_2 = 171$ ... (iii)

Subtracting (iii) from (ii) we have

$7f_2 - 3f_2 = 279 - 171$

$$\Rightarrow 4f_2 = 108 \Rightarrow f_2 = \frac{108}{4} = 27$$

Now, put $f_2 = 27$ in (i), we have

$f_1 + 27 = 57$

$f_1 = 57 - 27 = 30$

Hence, $f_1 = 30$ and $f_2 = 27$

Assertion and Reason Answers-

1. (d) Assertion is wrong statement but reason is correct statement.

Explanation:

Know that the first 5 multiples of 4 are

⇒ 4, 8, 12, 16, 20

The range is given as the difference between the maximum value and the minimum value.

Therefore, the range of multiples of 4 is

⇒ 20 – 4

⇒ 16

Hence, (A) is wrong but (R) is true.

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation:

Number of terms = 10 (even)

Median of n even number of terms

$$= \frac{\left(\frac{n}{2}\right)^{th}\text{ term} + \left(\frac{n}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{5^{th}\text{ term} + 6^{th}\text{ term}}{2} = \frac{x+x+2}{2} = 63$$

$\Rightarrow 2x + 2 = 126$

$\Rightarrow x = 62$

Probability 15

1. **Probability** is a quantitative measure of certainty.
2. Any activity associated to certain outcome is called an experiment.
   For example: (i) tossing a coin (ii) throwing a dice (iii) selecting a card.
3. An outcome is a result of a single trial of an experiment.
   For example: two possible outcomes of tossing a coin are head and tail.
4. An event for an experiment is the collection of some outcomes of the experiment.
   For example: (i) Getting a head on tossing a coin (ii) getting a face card when a card is drawn from a pack of 52 cards.
5. The empirical (experimental) probability of an event E denoted as P(E) is given by:
   $$P(E) = \frac{\text{Number pf trials in which the event happened}}{\text{Total number of outcomes}}$$
6. Probability of an event lies between 0 and 1. Probability can never be negative.

Probability

Probability is the measure of the likelihood of an event to occur. Events can't be predicted with certainty but can be expressed as to how likely it can occur using the idea of probability.

Probability can range between 0 and 1, where 0 probability means the event to be an impossible one and probability of 1 indicates a certain event.

Probability means possibility. It is a branch of mathematics that deals with the occurrence of a random event. The value is expressed from zero to one. Probability has been introduced in Maths to predict how likely events are to happen. The meaning of probability is basically the extent to which something is likely to happen. This is the basic probability theory, which is also used in the probability distribution, where you will learn the possibility of outcomes for a random experiment. To find the probability of a single event to occur, first, we should know the total number of possible outcomes.

Formula for Probability

The probability formula is defined as the possibility of an event to happen is equal to the ratio of the number of favourable outcomes and the total number of outcomes.

Probability of event to happen P(E) = Number of favourable outcomes/Total Number of outcomes

Sometimes students get mistaken for "favourable outcome" with "desirable outcome". This is the basic formula. But there are some more formulas for different situations or events.

Solved Examples

1) There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?

Ans: The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows, i.e. 2/6 = 1/3.

2) There is a container full of coloured bottles, red, blue, green and orange. Some of the bottles are picked out and displaced. Sumit did this 1000 times and got the following results:

No. of blue bottles picked out: 300

No. of red bottles: 200

No. of green bottles: 450

No. of orange bottles: 50

a) What is the probability that Sumit will pick a green bottle?

Ans: For every 1000 bottles picked out, 450 are green.

Therefore, P(green) = 450/1000 = 0.45

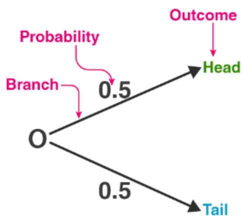
b) If there are 100 bottles in the container, how many of them are likely to be green?

Ans: The experiment implies that 450 out of 1000 bottles are green.

Therefore, out of 100 bottles, 45 are green.

Probability Tree

The tree diagram helps to organize and visualize the different possible outcomes. Branches and ends of the tree are two main positions. Probability of each branch is written on the branch, whereas the ends are containing the final outcome. Tree diagrams are used to figure out when to multiply and when to add. You can see below a tree diagram for the coin:



Types of Probability

There are three major types of probabilities:

Theoretical Probability

Experimental Probability

Axiomatic Probability

Theoretical Probability

It is based on the possible chances of something to happen. The theoretical probability is mainly based on the reasoning behind probability. For example, if a coin is tossed, the theoretical probability of getting a head will be ½.

Experimental Probability

It is based on the basis of the observations of an experiment. The experimental probability can be calculated based on the number of possible outcomes by the total number of trials. For example, if a coin is tossed 10 times and heads is recorded 6 times then, the experimental probability for heads is 6/10 or, 3/5.

Axiomatic Probability

In axiomatic probability, a set of rules or axioms are set which applies to all types. These axioms are set by Kolmogorov and are known as Kolmogorov's three axioms. With the axiomatic approach to probability, the chances of occurrence or non-occurrence of the events can be quantified. The axiomatic probability lesson covers this concept in detail with Kolmogorov's three rules (axioms) along with various examples.

Conditional Probability is the likelihood of an event or outcome occurring based on the occurrence of a previous event or outcome.

Probability of an Event

Assume an event E can occur in r ways out of a sum of n probable or possible equally likely ways. Then the probability of happening of the event or its success is expressed as;

P(E) = r/n

The probability that the event will not occur or known as its failure is expressed as:

P(E') = (n–r)/n = 1–(r/n)

E' represents that the event will not occur.

Therefore, now we can say;

P(E) + P(E') = 1

This means that the total of all the probabilities in any random test or experiment is equal to 1.

Equally Likely Events

When the events have the same theoretical probability of happening, then they are called equally likely events. The results of a sample space are called equally likely if all of them have the same probability of occurring. For example, if you throw a die, then the probability of getting 1 is 1/6. Similarly, the probability of getting all the numbers from 2,3,4,5 and 6, one at a time is 1/6. Hence, the following are some examples of equally likely events when throwing a die:

- Getting 3 and 5 on throwing a die
- Getting an even number and an odd number on a die
- Getting 1, 2 or 3 on rolling a die

are equally likely events, since the probabilities of each event are equal.

Complementary Events

The possibility that there will be only two outcomes which states that an event will occur or not. Like a person will come or not come to your house, getting a job or not getting a job, etc. are examples of complementary events. Basically, the complement of an event occurring in the exact opposite that the probability of it is not occurring. Some more examples are:

It will rain or not rain today

The student will pass the exam or not pass.

You win the lottery or you don't.

Probability Theory

Probability theory had its root in the 16th century when J. Cardan, an Italian mathematician and physician, addressed the first work on the topic, The Book on Games of Chance. After its inception, the knowledge of probability has brought to the attention of great mathematicians. Thus, Probability theory is the branch of mathematics that deals with the possibility of the happening of events. Although there are many distinct probability interpretations, probability theory interprets the concept precisely by expressing it through a set of axioms or hypotheses. These hypotheses help form the probability in terms of a possibility space, which allows a measure holding values between 0 and 1. This is known as the probability measure, to a set of possible outcomes of the sample space.

Probability Density Function

The Probability Density Function (PDF) is the probability function which is represented for the density of a continuous random variable lying between a certain range of values. Probability Density Function explains the normal distribution and how mean and deviation exists. The standard normal distribution is used to create a database or statistics, which are often used in science to represent the real-valued variables, whose distribution is not known.

Experiment

An experiment:

is any procedure that can be infinitely repeated or any series of actions that have a well-defined set of possible outcomes.

can either have only one or more than one possible outcome. is also called the sample space.

Trail

A single event that is performed to determine the outcome is called a trial.

All possible trials that constitute a well-defined set of possible outcomes are collectively called an experiment/ sample space.

Experimental Probability

Experimental/Empirical Probability

The empirical probability of an event that may happen is given by:

Probability of event to happen P(E)=Number of favourable outcomes/Total number of outcomes

You and your 3 friends are playing a board game. It's your turn to roll the die and to win the game you need a 5 on the dice. Now, is it possible that upon rolling the die you will get an exact 5? No, it is a matter of chance. We face multiple situations in real life where we have to take a chance or risk. Based on certain conditions, the chance of occurrence of a certain event can be easily predicted. In our day to day life, we are more familiar with the word 'chance and probability'. In simple words, the chance of occurrence of a particular event is what we study in probability. In this article, we are going to discuss one of the types of probability called "Experimental Probability" in detail.

Experimental Probability Vs Theoretical Probability

There are two approaches to study probability:

- Experimental Probability
- Theoretical Probability

Experimental Probability

Experimental probability, also known as Empirical probability, is based on actual experiments and adequate recordings of the happening of events. To determine the occurrence of any event, a series of actual experiments are conducted. Experiments which do not have a fixed result are known as random experiments. The outcome of such experiments is uncertain. Random experiments are repeated multiple times to determine their likelihood. An experiment is repeated a fixed number of times and each repetition is known as a trial. Mathematically, the formula for the experimental probability is defined by;

Probability of an Event P(E) = Number of times an event occurs / Total number of trials.

Theoretical Probability

In probability, the theoretical probability is used to find the probability of an event. Theoretical probability does not require any experiments to conduct. Instead of that, we should know about the situation to find the probability of an event occurring. Mathematically, the theoretical probability is described as the number of favourable outcomes divided by the number of possible outcomes.

Probability of Event P(E) = No. of. Favourable outcomes/ No. of. Possible outcomes.

Example: You asked your 3 friends Shakshi, Shreya and Ravi to toss a fair coin 15 times each in a row and the outcome of this experiment is given as below:

Coin Tossed By:	No. of. Heads	No. of. Tails
Sakshi	6	9
Shreya	7	8
Ravbi	8	7

Calculate the probability of occurrence of heads and tails.

Solution: The experimental probability for the occurrence of heads and tails in this experiment can be calculated as:

Experimental Probability of Occurrence of heads = Number of times head occurs/Number of times coin is tossed.

Experimental Probability of Occurrence of tails = Number of times tails occurs/Number of times coin is tossed.

Coin Tossed By:	No. of. Heads	No. of. Tails	Experimental Probability for the occurrence of Head	Experimental Probability for the occurrence of Tail
Sakshi	6	9	6/15 = 0.4	9/15 = 0.6
Shreya	7	8	7/15 = 0.47	8/15 = 0.53
Ravbi	8	7	8/15 = 0.53	7/15 = 0.47

We observe that if the number of tosses of the coin increases then the probability of occurrence of heads or tails also approaches to 0.5.

Coin Tossing Experiment

Consider a fair coin. There are only two possible outcomes that are either getting heads or tails.

Number of possible outcomes = 2

Number of outcomes to get head = 1

The probability of getting head = Number of outcomes to get head/Number of possible outcomes = ½

Rolling of Dice Experiment

When a fair dice is rolled, the number that comes up top is a number between one to six. Assuming we roll the dice once, to check the possibility of three coming up.

Number of possible outcomes = 6

Number of outcomes to get three = 1

The probability of getting three = Number of outcomes to get three/Number of possible outcomes=1/6

Sum of Probabilities of Favorable and Unfavourable events

- When a trial is done for an expected outcome, there are chances when the expected outcome is achieved. Such a trial/event is called a favourable event.
- When a trial is done for an expected outcome, there are chances when the expected outcome is not achieved. Such a trial/event is called an unfavourable event.
- All favourable and unfavourable event outcomes come from the well-defined set of outcomes.
- Suppose an event of sample space S has n favourable outcomes. Then, there are S-n, unfavourable outcomes.
- The probability of favourable and unfavourable events happening depends upon the number of trials performed. However, the sum of both these probabilities is always equal to one.

Class : 9th mathematics
Chapter- 15: Probability

Probability

- **Definition**
  - The extent to which something is likely to happen
  - Impossible — Even chance — Certain; ← Less likely, More likely →
- **Related terms**
  - **Outcome**: Something that follows as a result or consequence
  - **Event**: Collection of some outcomes of an experiment
  - **Trial**: Action which results in one or several outcomes
- **Empirical probability**
  - $P(E) = \dfrac{\text{Number of trials in which event happened}}{\text{Total number of trials}}$
- **Probability value**
  - Never greater than one 0<P(E)<1
  - Never be negative
- **Examples**
  - **Coin**

A coin is tossed 3 times. Find probability of three heads?

Total number of trials = 2^n, n is number of times coin is tossed

$= 2^3$

$= 8$

i.e. if n=3{HHH, HHT, HTT, HTH, TTH, THH, TTT, THT}

$P(E) = \dfrac{\text{Number of times three heads occured}}{\text{Total number of trials}}$

$= \dfrac{1}{8}$

  - **Dice**

A dice is thrown 2 times. Find the probability of getting 11

Total number of trials = 6^n, n is the number of times dice is thrown

$= 6^2 = 36$

Outcomes for getting 11 = (5,6) (6,5)

$P(E) = \dfrac{\text{Number of getting 11}}{\text{Total number of trials}} = \dfrac{2}{36}$

$= \dfrac{1}{18}$

Important Questions

Multiple Choice Questions

1. Which of the following cannot be the empirical probability of an event?
   (a) $\frac{2}{3}$
   (b) $\frac{3}{2}$
   (c) 0
   (d) 1
2. In a survey of 364 children aged 19-36 months, it was found that 91 liked to eat potato chips. If a child is selected at random, the probability that he/she does not like to eat potato chips is:
   (a) 0.25
   (b) 0.50
   (c) 0.75
   (d) 0.80
3. In a sample study of 640 people, it was found that 512 people have a high school certificate. If a person is selected at random, the probability that the person has a high school certificate is:
   (a) 0.5
   (b) 0.6
   (c) 0.7
   (d) 0.8
4. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is
   (a) $\frac{4}{15}$
   (b) $\frac{2}{15}$
   (c) $\frac{1}{5}$
   (d) $\frac{1}{3}$
5. When a die is thrown, the probability of getting an odd number less than 4 is
   (a) $\frac{1}{6}$
   (b) $\frac{1}{3}$
   (c) $\frac{1}{2}$
   (d) 0
6. A bag contains 16 cards bearing number 1, 2, 3 ...., 16 respectively. One card is drawn at random. What is the probability that a number is divisible by 3?
   (a) $\frac{3}{16}$
   (b) $\frac{5}{16}$
   (c) $\frac{11}{16}$
   (d) $\frac{13}{16}$
7. In a cricket match a batsman hits a boundary 4 times out of the 32 balls he plays. In a given ball, what is the probability that he does not hit the ball to the boundary?
   (a) $\frac{7}{8}$
   (b) $\frac{1}{8}$
   (c) $\frac{1}{7}$
   (d) $\frac{6}{7}$
8. The sum of the probabilities of all events of a trial is
   (a) 1
   (b) Greater than 1
   (c) Less than 1
   (d) Between 0 and 1
9. A die is thrown 300 times and odd numbers are obtained 153 times. Then the probability of getting an even number is
   (a) $\frac{153}{300}$
   (b) $\frac{147}{300}$
   (c) $\frac{174}{300}$
   (d) $\frac{147}{153}$
10. Two coins are tossed 1000 times and the outcomes are recorded as below:

No. of heads	2	1	0
Frequency	200	550	250

The probability of getting at the most one head is:

(a) $\frac{1}{5}$

(b) $\frac{1}{4}$

(c) $\frac{4}{5}$

(d) $\frac{3}{4}$

Very Short Questions:

1. The blood groups of some students of Class IX were surveyed and recorded as below:

Blood Group	A	B	AB	O
No. of students	19	6	13	12

If a student is chosen at random, find the probability that he/she has blood group A or AB.

2. A group of 80 students of Class X are selected and asked for their choice of subject to be taken in Class XI, which is recorded as below:

Stream	PCM	PCB	Commerce	Humanities	Total
No. of students	29	18	21	12	80

If a student is chosen at random, find the probability that he/she is a student of either commerce or humanities stream.

3. A box contains 50 bolts and 150 nuts. On checking the box, it was found that half of the bolts and half of the nuts are rusted. If one item is chosen at random, find the probability that it is rusted.

4. A dice is rolled number of times and its outcomes are recorded as below:

Outcome	1	2	3	4	5	6
Frequency	35	45	50	38	53	29

Find the probability of getting an odd number.

5. The probability of guessing the correct answer to a certain question is x^2. If probability of not guessing the correct answer is 2/3, then find x.

6. A bag contains x white, y red and z blue balls. A ball is drawn at the random, then what is the probability of drawing a blue ball.

Short Questions:

1. 750 families with 3 children were selected randomly and the following data recorded:

No. of girls in a family	0	1	2	3
No. of families	120	220	310	100

If a family member is chosen at random, compute the probability that it has:

(i) no boy child

(ii) no girl child

2. If the probability of winning a race of an athlete is $\frac{1}{6}$ less than the twice the probability of losing the race. Find the probability of winning the race.

3. Three coins are tossed simultaneously 150 times with the following frequencies of different outcomes:

No. of tails	0	1	2	3
Frequency	25	30	32	63

Compute the probability of getting:

(i) At least 2 tails

(ii) Exactly one tail

4. The table shows the marks obtained by a student in unit tests out of 50.

Unit Test	I	II	III	IV	V
Marks (Out of 50)	34	35	36	34	37

Find the probability that the student gets 70% or more in the next unit test. Also, the probability that student get less than 70%.

5. Books are packed in piles each containing 20 books. Thirty-five piles were examined for defective books and the results are given in the following table:

Long Questions:

1. Three coins are tossed simultaneously 250 times. The distribution of various outcomes is listed below:

(i) Three tails: 30,

(ii) Two tails: 70,

(iii) One tail: 90,

(iv) No tail: 60

Find the respective probability of each event and check that the sum of all probabilities is

2. A travel company has 100 drivers for driving buses to various tourist destination. Given below is a table showing the resting time of the drivers after covering a certain distance (in km).

Distance (in km)	After 80 km	After 115 km	After 155 km	After 200 km
No. of drivers	19	6	13	12

What is the probability that the driver was chosen at random?

(a) takes a halt after covering 80km.

(b) takes a halt after covering 115km.

(c) takes a halt after covering 155km.

(d) takes a halt after crossing 200km.

3. A company selected 2300 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a home. The information gathered is listed in the table below:

Monthly Income (in ₹)	Vehicles per Family			
	0	1	2	Above 2
Less than 7000	10	140	25	0
7000 – 10000	0	295	27	12
10000 – 13000	1	525	39	11
13000 – 16000	2	449	29	25
16000 or more	1	539	82	88

If a family is chosen at random, find the probability that the family is:

(i) earning ₹7000 – ₹13000 per month and owning exactly 1 vehicle.

(ii) owning not more than one vehicle. (iii) earning more than ₹13000 and owning 2 or more than 2 vehicles. (iv) owning no vehicle

4. A survey of 2000 people of different age groups was conducted to find out their preference in watching different types of movies:

Type I + Family Type II → Comedy and Family

Type III → Romantic, Comedy, and Family 242.

Type IV → Action, Romantic, Comedy and Family

Age group	Type I	Type II	Type III	Type IV	All
18 – 29	440	160	110	61	35
30 – 50	505	125	60	22	18
Above 50	360	45	35	15	9

Find the probability that a person chosen at random is:

(a) in 18-29 years of age and likes type II movies

(b) above 50 years of age and likes all types of movies

(c) in 30-50 years and likes type I movies.

5. 5. In a kitchen, there are 108 utensils, consisting of bowls, plates, and glasses. The ratio of bowls, plates the glasses is 4:2:3. A utensil is picked at random. Find the probability that:

(i) it is a plate.

(ii) it is not a bowl.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

   c) Assertion is correct statement but reason is wrong statement.

   d) Assertion is wrong statement but reason is correct statement.

   Assertion: A die is thrown. Let E be the event that number appears on the upper face is less than 1, then P (E) = 1/6

   Reason: Probability of impossible event is 0.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

   a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

   b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

   c) Assertion is correct statement but reason is wrong statement.

   d) Assertion is wrong statement but reason is correct statement.

   Assertion: A coin is tossed two times. Probability of getting at least two heads is 1/6.

   Reason: When a coin is tossed two times, then the sample space is {HH, HT, TH, TT}.

Case Study Questions:

1. Read the Source/ Text given below and answer these questions:

   Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes given in the table. Read the data given in the table carefully.

The blue colour was painted by Deepak, Black by Suresh, green by Harsh and the yellow was painted by Naresh.

Outcome	3 tails	2 tails	1 tail	no tail
Frequency	19	6	13	12

If the three coins are simultaneously tossed again, compute the probability of:

i. Getting less than 3 tails:
   a. 0.9
   b. 0.1
   c. 0.01
   d. 0.02

ii. Exactly 2 Heads:
   a. 0.68
   b. 0.41
   c. 0.34
   d. 0.5

iii. Exactly 1 head:
   a. 0.68
   b. 0.86
   c. 0.34
   d. 0.11

iv. At least 1 tail:
   a. 0.58
   b. 0
   c. 1
   d. 0.85

v. All heads:
   a. 0.51
   b. 0.55
   c. 0.9
   d. 0.15

2. Read the Source/ Text given below and answer any four questions:

Over the past 200 working days, the number of defective parts produced by a machine in a factory is given in the following table:

No. of defective parts	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Days	50	32	22	18	12	12	10	10	10	8	6	6	2	2

Determine the probability that tomorrow's output will have.

i. No. defective part
   a. 0.25
   b. 0
   c. 0.50
   d. 0.025

ii. At least one defective part
   a. 0.50
   b. 0.75
   c. 0.32
   d. 0.01

iii. Not more than 5 defective parts
   a. 0.12
   b. 0.75
   c. 0.73
   d. 0.60

iv. More than 13 defective parts
   a. 0
   b. 1
   c. −1
   d. 0.2

v. At most 3 defective parts
   a. −0.12
   b. 0.50
   c. 0.18
   d. 0.61

Answer Key

Multiple Choice Questions

1. (b) $\frac{3}{2}$
2. (c) 0.75
3. (d) 0.8
4. (c) $\frac{1}{5}$
5. (b) $\frac{1}{3}$
6. (b) $\frac{5}{16}$
7. (a) $\frac{7}{8}$
8. (a) 1
9. (b) $\frac{147}{300}$
10. (c) $\frac{4}{5}$

Very Short Answer:

1. Here,
total number of students = 19 + 6 + 13 + 12 = 50
Number of students has blood group A or AB
= 19 + 13 = 32
Required probability = $\frac{38}{50} = \frac{16}{25}$

2. Here, total number of students = 80
Total number of students of Commerce or Humanities stream = 33
Required probability = $\frac{33}{80}$

3. Total number of nuts and bolts in the box
= 150 + 50 = 200
Number of nuts and bolts rusted = ½ × 200 = 100
P(a rusted nut or bolt) = $\frac{100}{200} = \frac{1}{2}$

4. Total number of outcomes = 250
Total number of outcomes of getting odd numbers = 35 + 50 + 53 = 138
∴ P(getting an odd number) = $\frac{138}{250} = \frac{69}{125}$

5. Here, probability of guessing the correct answer
$= \frac{x}{2}$
And probability of not guessing the correct answer = $\frac{x}{2}$
Now, $\frac{x}{2} + \frac{2}{3} = 1$
$\Rightarrow 3x + 4 = 6$
$\Rightarrow 3x = 2$
$\Rightarrow x = \frac{2}{3}$

6. Number of blue balls = Z
Total balls = x + y + Z
∴ P(a blue ball) = $\frac{z}{x+y+z}$
Hence, new median is 34.

Short Answer:

1. (i) P(no boy child) = $\frac{100}{750} = \frac{2}{15}$
and P (no girl child) = $\frac{120}{750} = \frac{4}{25}$

2. Let probability of winning the race be p
∴ Probability of losing the race = 1– p
According to the statement of question, we have
$p = 2\,(1 - p) - \frac{1}{6}$
$\Rightarrow 6p = 12 - 12p - 1$
$\Rightarrow 18p = 11$
$\Rightarrow p = \frac{11}{18}$.
Hence, probability of winning the race is $\frac{11}{18}$.

3. Here, total number of chances = 150
(i) Total number of chances having at least 2 tails
= 32 + 63 = 95
∴ Required probability = $\frac{95}{150} = \frac{19}{30}$.

(ii) Total number of chances having exactly one tail = 30

∴ Required probability = $\frac{30}{150} = \frac{1}{5}$.

4. Here, the marks are out of 50, so we first find its percentage (i.e., out of 100)

Unit Test	I	II	III	IV	V
Marks (Out of 50)	68	70	72	68	74

Total number of outcomes = 5

Probability of getting 70% or more marks = $\frac{3}{5}$.

Probability of getting less than 70% = $\frac{2}{5}$.

5. Total number of books = 700

(i) P(no defective books) = $\frac{400}{700} = \frac{4}{7}$

(ii) P(more than 0 but less than 4 defective books) = $\frac{269}{700}$

13 (iii) P(more than 4 defective books) = $\frac{13}{700}$

Long Answer:

1. Here, the total number of chances = 250

Total number of three tails = 30

∴ P(of three tails) = $\frac{30}{250} = \frac{3}{25}$

(ii) Total number of two tails = 70

∴ P(of two tails) = $\frac{70}{250} = \frac{7}{25}$

(iii) Total number of one tail = 90

∴ P(of one tail) = $\frac{90}{250} = \frac{9}{25}$

(iv) Total number of no tail = 60

∴ P(of no tail) = $\frac{60}{250} = \frac{6}{25}$

Now, sum of all probabilities

$= \frac{3}{25} + \frac{7}{25} + \frac{9}{25} + \frac{6}{25} = \frac{25}{25} = 1$

2. Total number of drivers = 100

(a) P (takes a halt after covering 80km) = $\frac{13}{100}$

(b) P (takes a halt after covering 115km)

$= \frac{60}{100} = \frac{3}{5}$

(c) P (takes a halt after covering 155km)

$= \frac{90}{100} = \frac{9}{10}$

(d) P (takes a halt after covering 200km)

$= \frac{10}{100} = \frac{1}{10}$

3. Here, we have a total number of families = 2300

(i) Number of families earning ₹ 7000 to ₹ 13000 per month and owning exactly 1 vehicle

= 295 + 525 = 820

∴ Required probability = $\frac{820}{2300} = \frac{41}{115}$.

(ii) Number of families owing more than one vehicle = 1962

∴ Required probability = $\frac{1962}{2300} = \frac{981}{1150}$.

(iii) Number of families earning more than ₹ 13000 one owning 2 or more than 2 vehicles = 224

∴ Required probability = $\frac{224}{2300} = \frac{56}{575}$.

(iv) Number of families owing no vehicle = 14

∴ Required probability = $\frac{14}{2300} = \frac{7}{1150}$.

4. (a) Let E_1 be the event, between the age group (18 - 29) years and liking type II movies

Favorable outcomes to event E_1 = 160

∴ $P(E_1) = \frac{160}{2000} = \frac{160}{2000}$

(b) Let E_2 be the event, of age group above 50 years and like all types of movies

Favorable outcomes to event E_2 = 9

∴ $P(E_2) = \frac{9}{2000}$

(c) Let E_3 be the event, between age group (30 - 50) years and liking type I movies

Favorable outcomes to event E_3 = 505

∴ $P(E_3) = \frac{505}{2000} = \frac{101}{400}$

5. Total utensils in the kitchen = 108

Let number of bowls be 4x, number of plates be 2x and number of glasses be 3x

∴ 4x + 2x + 3x = 108

9x = 108

$$x = \frac{108}{9} = 12$$

Thus, number of bowls = 4 × 12 = 48

Number of plates = 2 × 12 = 24

Number of glasses = 3 × 12 =

(i) P (a plate) = $\frac{24}{100} = \frac{2}{9}$

(ii) P (not a bowl) = $\frac{24+36}{108} = \frac{60}{108} = \frac{5}{9}$

Assertion and Reason Answers-

1. (d) Assertion is wrong statement but reason is correct statement.

Explanation: When a die is thrown, then number of outcomes are 1, 2, 3, 4, 5, 6

P(number appear on the upper face is less than 1)=0

2. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: Number of total outcomes when a coin is tossed 2 times i.e., {HH, HT, TH, TT} = 4

P(getting at least two heads) = $\frac{1}{4}$

Case Study Questions:

1.

(i)	(a)	0.9
(ii)	(b)	0.41
(iii)	(d)	0.34
(iv)	(c)	0.85
(v)	(b)	0.15

2.

(i)	(b)	0.25
(ii)	(c)	0.75
(iii)	(d)	0.73
(iv)	(c)	0
(v)	(a)	0.61